BOHR MODEL FOR ISOELECTRONIC IONS

Exact energies of atomic ions were taken from E. R. Davidson's table available on-line at `http://php.indiana.edu/~davidson/atom/tub.4`.

For a given number of electrons N, and nuclear charge Z, the functional of energy is defined as

$$W = W_1 + W_2, \tag{1}$$

where

$$W_1 = \sum_{n=1}^{N} \left(\frac{Q(n)^2}{2R_n^2} - \frac{Z}{R_n} \right), \tag{2}$$

$$W_2 = \sum_{m=1}^{N-1} \sum_{n=m+1}^{N} \frac{1}{R_{mn}}, \tag{3}$$

$$R_n = \sqrt{x_n^2 + y_n^2 + z_n^2}, \tag{4}$$

$$R_{mn} = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2 + (z_m - z_n)^2}, \tag{5}$$

and

$$y_1 = z_1 = z_2 = 0. \tag{6}$$

In (2), $Q(n)$ for $n = 1, 2, 3, \ldots, N$ is a principal quantum number of nth electron (as an example for Li, $Q(1) = Q(2) = 1$, $Q(3) = 2$). The energy W is expressed by means of equations (1) - (6) through Cartesian coordinates $x_1, x_2, y_2, x_3, y_3, z_3, x_4, y_4, z_4, \ldots, x_N, y_N, z_N$, in total 1 coordinate if $N = 1$, 3 coordinates if $N = 2$, or $3N - 3$ coordinates if $N \geq 3$. Then, the energy is minimized in respect to these coordinates.

Calculations were done by the following *Mathematica* program, that accepts two arguments, N and Z, and returns the energy and distances with tallies.

```
(* Bohr model for any atom or ion *)
<< Miscellaneous`ChemicalElements`;
bohr[npart_Integer, zz_Integer] := Module[
{name, shortname, config, conf, title, mshells, n, mels, qnumber, x, y, z, r, en1, en2, w, vars, enz, sol,
 rs, rads, mrads, radms, time},
If[!ValueQ[prec], prec = 128]; (* working precision *)
If[!ValueQ[maxit], maxit = 333]; (maximal number of iterations *)
name = Elements[[npart]];
shortname = Abbreviation[name];
config = ElectronConfigurationFormat[name] // TeXForm // ToString;
conf = Plus @@ # & /@ ElectronConfiguration[name];
title = ToString[name] <> " isoelectronic series - " <> ToString[npart] <> " electrons: ";
mshells = Length[conf];
n = 0;
Do[
    mels = conf[[nshell]];
    Do[
```

```
      n++;
      qnumber[n] = nshell, {mels}], {nshell, mshells}];
y[1] = z[1] = z[2] = 0;
en1 =
    Sum[
      r[n] = Sqrt[x[n]^2 + y[n]^2 + z[n]^2];
      -zz/r[n] + qnumber[n]^2/2/r[n]^2, {n, npart}];
en2 =
    Sum[Sum[
        r[n, m] = Sqrt[(x[n] - x[m])^2 + (y[n] - y[m])^2 + (z[n] - z[m])^2];
        1/r[n, m], {m, n + 1, npart}], {n, npart}];
w = en1 + en2;
vars = Table[{x[n], y[n], z[n]}, {n, npart}] // Flatten;
vars = Cases[vars, _[_]];
time = Timing[
{enz, sol} = NMinimize[{w}, vars,
      WorkingPrecision -> prec,
      MaxIterations -> maxit];
 ][[1]];
enz = N[enz];
rs = N[Table[r[n], {n, npart}] /. sol] // Sort;
rads = rs // Union;
mrads = Length[rads];
radms = Table[Length[Select[rs, # == rads[[nrad]] &]], {nrad, mrads}];
Print[title, "   Z = ", zz, "   R = ", {rads, radms} // TableForm, "   E = ",
    enz, "   ", time];
{enz, rads, radms}]
```

Results are printed in tables given below. Third column of each table lists the distances between electrons and the nucleus $R_1, R_2, R_3, \ldots, R_N$. If several distances are equal, they are listed as one number with the multiplicity shown in parenthesis. The fourth column gives result of the energy minimization E_{Bohr}, and the fifth column exact energy E_{exact}. The sixth (last) column shows an error as a percentage $100(E_{\text{Bohr}} - E_{\text{exact}})/E_{\text{exact}}$.

TABLE I: Helium isoelectronic series. Configuration $1s^2$. 2 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
1	H^-	1.333(2)	−0.5625	−0.5278	6.58
2	He	0.571(2)	−3.0625	−2.9037	5.47
3	Li^+	0.364(2)	−7.5625	−7.2799	3.88
4	Be^{2+}	0.267(2)	−14.0625	−13.6556	2.98
5	B^{3+}	0.211(2)	−22.5625	−22.031	2.41
6	C^{4+}	0.174(2)	−33.0625	−32.4062	2.03
7	N^{5+}	0.148(2)	−45.5625	−44.7814	1.74
8	O^{6+}	0.129(2)	−60.0625	−59.1566	1.53
9	F^{7+}	0.114(2)	−76.5625	−75.5317	1.36
10	Ne^{8+}	0.103(2)	−95.0625	−93.9068	1.23
11	Na^{9+}	0.093(2)	−115.5625	−114.2819	1.12
12	Mg^{10+}	0.085(2)	−138.0625	−136.6569	1.03
13	Al^{11+}	0.078(2)	−162.5625	−161.032	0.95
14	Si^{12+}	0.073(2)	−189.0625	−187.4071	0.88
15	P^{13+}	0.068(2)	−217.5625	−215.7821	0.83
16	S^{14+}	0.063(2)	−248.0625	−246.1571	0.77
17	Cl^{15+}	0.06(2)	−280.5625	−278.5322	0.73
18	Ar^{16+}	0.056(2)	−315.0625	−312.9072	0.69
19	K^{17+}	0.053(2)	−351.5625	−349.2822	0.65
20	Ca^{18+}	0.051(2)	−390.0625	−387.6572	0.62
21	Sc^{19+}	0.048(2)	−430.5625	−428.0323	0.59
22	Ti^{20+}	0.046(2)	−473.0625	−470.4073	0.56
23	V^{21+}	0.044(2)	−517.5625	−514.7823	0.54
24	Cr^{22+}	0.042(2)	−564.0625	−561.1573	0.52
25	Mn^{23+}	0.04(2)	−612.5625	−609.5323	0.5
26	Fe^{24+}	0.039(2)	−663.0625	−659.9073	0.48
27	Co^{25+}	0.037(2)	−715.5625	−712.2823	0.46
28	Ni^{26+}	0.036(2)	−770.0625	−766.6574	0.44

TABLE II: Lithium isoelectronic series. Configuration $1s^2 2s^1$. 3 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
3	Li	$0.364(2), 3.849$	-7.6905	-7.4781	2.84
4	Be^{+}	$0.267(2), 1.909$	-14.5767	-14.3248	1.76
5	B^{2+}	$0.211(2), 1.275$	-23.7185	-23.4246	1.25
6	C^{3+}	$0.174(2), 0.96$	-35.1134	-34.7755	0.97
7	N^{4+}	$0.148(2), 0.771$	-48.7602	-48.3769	0.79
8	O^{5+}	$0.129(2), 0.644$	-64.6581	-64.2285	0.67
9	F^{6+}	$0.114(2), 0.554$	-82.8067	-82.3303	0.58
10	Ne^{7+}	$0.103(2), 0.486$	-103.2058	-102.6822	0.51
11	Na^{8+}	$0.093(2), 0.433$	-125.8553	-125.2842	0.46
12	Mg^{9+}	$0.085(2), 0.39$	-150.755	-150.1362	0.41
13	Al^{10+}	$0.079(2), 0.356$	-177.9049	-177.2382	0.38
14	Si^{11+}	$0.073(2), 0.326$	-207.305	-206.5903	0.35
15	P^{12+}	$0.068(2), 0.302$	-238.9553	-238.1924	0.32
16	S^{13+}	$0.064(2), 0.28$	-272.8556	-272.0445	0.3
17	Cl^{14+}	$0.06(2), 0.262$	-309.006	-308.1466	0.28
18	Ar^{15+}	$0.056(2), 0.246$	-347.4064	-346.4987	0.26
19	K^{16+}	$0.053(2), 0.232$	-388.057	-387.1008	0.25
20	Ca^{17+}	$0.051(2), 0.219$	-430.9575	-429.953	0.23
21	Sc^{18+}	$0.048(2), 0.207$	-476.1081	-475.0551	0.22
22	Ti^{19+}	$0.046(2), 0.197$	-523.5088	-522.4073	0.21
23	V^{20+}	$0.044(2), 0.188$	-573.1595	-572.0094	0.2
24	Cr^{21+}	$0.042(2), 0.18$	-625.0602	-623.8616	0.19
25	Mn^{22+}	$0.04(2), 0.172$	-679.2109	-677.9638	0.18
26	Fe^{23+}	$0.039(2), 0.165$	-735.6116	-734.3159	0.18
27	Co^{24+}	$0.037(2), 0.158$	-794.2624	-792.9181	0.17
28	Ni^{25+}	$0.036(2), 0.152$	-855.1632	-853.7702	0.16

TABLE III: Beryllium isoelectronic series. Configuration $1s^2 2s^2$. 4 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
3	Li^-	$0.364(2), 5.233(2)$	-7.7049	-7.5008	2.72
4	Be	$0.267(2), 2.232(2)$	-14.8403	-14.6674	1.18
5	B^+	$0.211(2), 1.421(2)$	-24.4825	-24.3489	0.55
6	C^{2+}	$0.174(2), 1.044(2)$	-36.6286	-36.5349	0.26
7	N^{3+}	$0.148(2), 0.826(2)$	-51.2772	-51.2229	0.11
8	O^{4+}	$0.129(2), 0.683(2)$	-68.4273	-68.4118	0.02
9	F^{5+}	$0.114(2), 0.583(2)$	-88.0784	-88.1012	-0.03
10	Ne^{6+}	$0.103(2), 0.508(2)$	-110.2303	-110.291	-0.06
11	Na^{7+}	$0.093(2), 0.451(2)$	-134.8827	-134.981	-0.07
12	Mg^{8+}	$0.085(2), 0.405(2)$	-162.0355	-162.1711	-0.08
13	Al^{9+}	$0.079(2), 0.368(2)$	-191.6887	-191.8614	-0.09
14	Si^{10+}	$0.073(2), 0.337(2)$	-223.8421	-224.0517	-0.09
15	P^{11+}	$0.068(2), 0.31(2)$	-258.4956	-258.7421	-0.1
16	S^{12+}	$0.064(2), 0.288(2)$	-295.6494	-295.9325	-0.1
17	Cl^{13+}	$0.06(2), 0.269(2)$	-335.3033	-335.623	-0.1
18	Ar^{14+}	$0.056(2), 0.252(2)$	-377.4573	-377.8135	-0.09
19	K^{15+}	$0.053(2), 0.237(2)$	-422.1113	-422.5041	-0.09
20	Ca^{16+}	$0.051(2), 0.224(2)$	-469.2655	-469.6946	-0.09
21	Sc^{17+}	$0.048(2), 0.212(2)$	-518.9197	-519.3852	-0.09
22	Ti^{18+}	$0.046(2), 0.201(2)$	-571.074	-571.5758	-0.09
23	V^{19+}	$0.044(2), 0.191(2)$	-625.7283	-626.2664	-0.09
24	Cr^{20+}	$0.042(2), 0.183(2)$	-682.8827	-683.457	-0.08
25	Mn^{21+}	$0.04(2), 0.175(2)$	-742.5371	-743.1477	-0.08
26	Fe^{22+}	$0.039(2), 0.167(2)$	-804.6916	-805.3383	-0.08
27	Co^{23+}	$0.037(2), 0.161(2)$	-869.3461	-870.0289	-0.08
28	Ni^{24+}	$0.036(2), 0.154(2)$	-936.5006	-937.2196	-0.08

TABLE IV: Boron isoelectronic series. Configuration $1s^2 2s^2 2p^1$. 5 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
4	Be^-	$0.267(2), 2.758(3)$	-14.8313	-14.6674	1.12
5	B	$0.211(2), 1.618(3)$	-24.7936	-24.6539	0.57
6	C^+	$0.174(2), 1.146(3)$	-37.5131	-37.431	0.22
7	N^{2+}	$0.148(2), 0.888(3)$	-52.9872	-52.9664	0.04
8	O^{3+}	$0.129(2), 0.725(3)$	-71.2144	-71.2556	-0.06
9	F^{4+}	$0.115(2), 0.613(3)$	-92.1936	-92.2972	-0.11
10	Ne^{5+}	$0.103(2), 0.531(3)$	-115.9243	-116.0902	-0.14
11	Na^{6+}	$0.093(2), 0.469(3)$	-142.4061	-142.6342	-0.16
12	Mg^{7+}	$0.085(2), 0.419(3)$	-171.6388	-171.929	-0.17
13	Al^{8+}	$0.079(2), 0.379(3)$	-203.622	-203.9742	-0.17
14	Si^{9+}	$0.073(2), 0.346(3)$	-238.3558	-238.7698	-0.17
15	P^{10+}	$0.068(2), 0.319(3)$	-275.8399	-276.3157	-0.17
16	S^{11+}	$0.064(2), 0.295(3)$	-316.0744	-316.6119	-0.17
17	Cl^{12+}	$0.06(2), 0.275(3)$	-359.0592	-359.6582	-0.17
18	Ar^{13+}	$0.056(2), 0.257(3)$	-404.7942	-405.4547	-0.16
19	K^{14+}	$0.053(2), 0.242(3)$	-453.2793	-454.0012	-0.16
20	Ca^{15+}	$0.051(2), 0.228(3)$	-504.5147	-505.2979	-0.16
21	Sc^{16+}	$0.048(2), 0.215(3)$	-558.5002	-559.3447	-0.15
22	Ti^{17+}	$0.046(2), 0.204(3)$	-615.2357	-616.1415	-0.15
23	V^{18+}	$0.044(2), 0.194(3)$	-674.7214	-675.6884	-0.14
24	Cr^{19+}	$0.042(2), 0.185(3)$	-736.9572	-737.9853	-0.14
25	Mn^{20+}	$0.04(2), 0.177(3)$	-801.9431	-803.0323	-0.14
26	Fe^{21+}	$0.039(2), 0.17(3)$	-869.679	-870.8293	-0.13
27	Co^{22+}	$0.037(2), 0.163(3)$	-940.165	-941.3764	-0.13
28	Ni^{23+}	$0.036(2), 0.156(3)$	-1013.401	-1014.673	-0.13

TABLE V: Carbon isoelectronic series. Configuration $1s^2 2s^2 2p^2$. 6 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
6	C	$0.174(2), 1.294(4)$	-37.8156	-37.8168	-0.03
7	N^{+}	$0.148(2), 0.976(4)$	-53.9059	-54.0546	-0.28
8	O^{2+}	$0.129(2), 0.783(4)$	-72.9976	-73.275	-0.38
9	F^{3+}	$0.114(2), 0.654(4)$	-95.0916	-95.5014	-0.43
10	Ne^{4+}	$0.103(2), 0.562(4)$	-120.1873	-120.7315	-0.45
11	Na^{5+}	$0.093(2), 0.492(4)$	-148.2845	-148.964	-0.46
12	Mg^{6+}	$0.085(2), 0.438(4)$	-179.3828	-180.1983	-0.45
13	Al^{7+}	$0.079(2), 0.395(4)$	-213.4821	-214.4338	-0.44
14	Si^{8+}	$0.073(2), 0.359(4)$	-250.5822	-251.6703	-0.43
15	P^{9+}	$0.068(2), 0.33(4)$	-290.683	-291.9075	-0.42
16	S^{10+}	$0.064(2), 0.304(4)$	-333.7843	-335.1453	-0.41
17	Cl^{11+}	$0.06(2), 0.283(4)$	-379.8861	-381.3835	-0.39
18	Ar^{12+}	$0.056(2), 0.264(4)$	-428.9882	-430.6221	-0.38
19	K^{13+}	$0.053(2), 0.248(4)$	-481.0907	-482.861	-0.37
20	Ca^{14+}	$0.051(2), 0.233(4)$	-536.1935	-538.1002	-0.35
21	Sc^{15+}	$0.048(2), 0.22(4)$	-594.2966	-596.3395	-0.34
22	Ti^{16+}	$0.046(2), 0.209(4)$	-655.3998	-657.5791	-0.33
23	V^{17+}	$0.044(2), 0.198(4)$	-719.5033	-721.8188	-0.32
24	Cr^{18+}	$0.042(2), 0.189(4)$	-786.607	-789.0586	-0.31
25	Mn^{19+}	$0.04(2), 0.181(4)$	-856.7107	-859.2985	-0.3
26	Fe^{20+}	$0.039(2), 0.173(4)$	-929.8147	-932.5385	-0.29
27	Co^{21+}	$0.037(2), 0.166(4)$	-1005.919	-1008.779	-0.28
28	Ni^{22+}	$0.036(2), 0.159(4)$	-1085.023	-1088.019	-0.28

TABLE VI: Nitrogen isoelectronic series. Configuration $1s^2 2s^2 2p^3$. 7 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
7	N	$0.148, 0.148, 1.052(2), 1.061(2), 1.159$	-54.161	-54.5893	-0.78
8	O^{+}	$0.129, 0.129, 0.835(2), 0.836(2), 0.895$	-73.928	-74.5669	-0.86
9	F^{2+}	$0.114, 0.114, 0.691(4), 0.732$	-96.947	-97.808	-0.88
10	Ne^{3+}	$0.103, 0.103, 0.588(4), 0.62$	-123.2169	-124.307	-0.88
11	Na^{4+}	$0.093, 0.093, 0.513(4), 0.537$	-152.7374	-154.0612	-0.86
12	Mg^{5+}	$0.085, 0.085, 0.454(4), 0.474$	-185.5084	-187.0689	-0.83
13	Al^{6+}	$0.079(2), 0.408(2), 0.408, 0.415(2)$	-221.5306	-223.3292	-0.81
14	Si^{7+}	$0.073(2), 0.37(2), 0.37, 0.376(2)$	-260.8048	-262.8414	-0.77
15	P^{8+}	$0.068(2), 0.338(2), 0.339, 0.344(2)$	-303.3297	-305.605	-0.74
16	S^{9+}	$0.064(2), 0.312(2), 0.312, 0.316(2)$	-349.1051	-351.6199	-0.72
17	Cl^{10+}	$0.06(2), 0.289(2), 0.29, 0.293(2)$	-398.131	-400.8856	-0.69
18	Ar^{11+}	$0.056(2), 0.27(2), 0.27, 0.273(2)$	-450.4073	-453.402	-0.66
19	K^{12+}	$0.053(2), 0.253(2), 0.253, 0.256(2)$	-505.934	-509.1691	-0.64
20	Ca^{13+}	$0.051(2), 0.238(2), 0.238, 0.24(2)$	-564.711	-568.1866	-0.61
21	Sc^{14+}	$0.048(2), 0.224(2), 0.225, 0.227(2)$	-626.7383	-630.4546	-0.59
22	Ti^{15+}	$0.046(2), 0.212(2), 0.213, 0.214(2)$	-692.0158	-695.973	-0.57
23	V^{16+}	$0.044(2), 0.202(2), 0.202, 0.203(2)$	-760.5436	-764.7416	-0.55
24	Cr^{17+}	$0.042(2), 0.192(2), 0.192, 0.194(2)$	-832.3215	-836.7605	-0.53
25	Mn^{18+}	$0.04(2), 0.183(2), 0.183, 0.185(2)$	-907.3496	-912.0296	-0.51
26	Fe^{19+}	$0.039(2), 0.175(2), 0.175, 0.177(2)$	-985.6278	-990.5489	-0.5
27	Co^{20+}	$0.037(2), 0.168(2), 0.168, 0.169(2)$	-1067.156	-1072.318	-0.48
28	Ni^{21+}	$0.036(2), 0.161(2), 0.161, 0.162(2)$	-1151.935	-1157.338	-0.47

TABLE VII: Oxygen isoelectronic series. Configuration $1s^2 2s^2 2p^4$. 8 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
8	O	$0.129(2), 0.878(4), 1.027(2)$	-74.178	-75.0674	-1.18
9	F^{+}	$0.114(2), 0.726(4), 0.8(2)$	-97.9282	-99.093	-1.18
10	Ne^{2+}	$0.103(2), 0.63(6)$	-125.1863	-126.6373	-1.15
11	Na^{3+}	$0.093(2), 0.544(6)$	-155.9469	-157.6923	-1.11
12	Mg^{4+}	$0.085(2), 0.479(6)$	-190.2084	-192.2543	-1.06
13	Al^{5+}	$0.078(2), 0.428(6)$	-227.9705	-230.3214	-1.02
14	Si^{6+}	$0.073(2), 0.386(6)$	-269.2334	-271.8921	-0.98
15	P^{7+}	$0.068(2), 0.352(6)$	-313.9967	-316.9657	-0.94
16	S^{8+}	$0.064(2), 0.324(6)$	-362.2605	-365.5414	-0.9
17	Cl^{9+}	$0.06(2), 0.3(6)$	-414.0247	-417.6188	-0.86
18	Ar^{10+}	$0.056(2), 0.279(6)$	-469.2893	-473.1976	-0.83
19	K^{11+}	$0.053(2), 0.26(6)$	-528.0541	-532.2776	-0.79
20	Ca^{12+}	$0.051(2), 0.245(6)$	-590.3193	-594.8585	-0.76
21	Sc^{13+}	$0.048(2), 0.23(6)$	-656.0846	-660.9402	-0.73
22	Ti^{14+}	$0.046(2), 0.218(6)$	-725.3502	-730.5225	-0.71
23	V^{15+}	$0.044(2), 0.207(6)$	-798.1159	-803.6054	-0.68
24	Cr^{16+}	$0.042(2), 0.196(6)$	-874.3818	-880.1888	-0.66
25	Mn^{17+}	$0.04(2), 0.187(6)$	-954.1478	-960.2726	-0.64
26	Fe^{18+}	$0.039(2), 0.179(6)$	-1037.414	-1043.857	-0.62
27	Co^{19+}	$0.037(2), 0.171(6)$	-1124.18	-1130.941	-0.6
28	Ni^{20+}	$0.036(2), 0.164(6)$	-1214.447	-1221.526	-0.58

TABLE VIII: Fluorine isoelectronic series. Configuration $1s^2 2s^2 2p^5$. 9 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
9	F	0.114,0.114,0.716(2),0.735(2),0.802,0.805,3.656	−98.059	−99.7341	−1.68
10	Ne^{+}	0.103(2),0.661(5),0.704(2)	−125.9247	−128.1437	−1.73
11	Na^{2+}	0.093(2),0.567(5),0.598(2)	−157.6955	−160.3258	−1.64
12	Mg^{3+}	0.085(2),0.497(5),0.52(2)	−193.2168	−196.2715	−1.56
13	Al^{4+}	0.078(2),0.442(5),0.46(2)	−232.4881	−235.9763	−1.48
14	Si^{5+}	0.073(2),0.398(2),0.398(2),0.399,0.412,0.412	−275.5098	−279.4376	−1.41
15	P^{6+}	0.068(2),0.362,0.362(2),0.363(2),0.372(2)	−322.2825	−326.6538	−1.34
16	S^{7+}	0.064(2),0.332,0.332(2),0.333(2),0.34(2)	−372.8059	−377.6237	−1.28
17	Cl^{8+}	0.06(2),0.306,0.307(2),0.308(2),0.313(2)	−427.0799	−432.3465	−1.22
18	Ar^{9+}	0.056(2),0.285(2),0.285,0.286(2),0.29(2)	−485.1044	−490.8217	−1.16
19	K^{10+}	0.053(2),0.266,0.266(2),0.267(2),0.27(2)	−546.8792	−553.0488	−1.12
20	Ca^{11+}	0.051(2),0.249,0.25(2),0.25(2),0.253(2)	−612.4044	−619.0275	−1.07
21	Sc^{12+}	0.048(2),0.234,0.235(2),0.236(2),0.238(2)	−681.6798	−688.7576	−1.03
22	Ti^{13+}	0.046(2),0.221,0.222(2),0.222(2),0.225(2)	−754.7055	−762.2387	−0.99
23	V^{14+}	0.044(2),0.21,0.21(2),0.211(2),0.213(2)	−831.4815	−839.4708	−0.95
24	Cr^{15+}	0.042(2),0.199,0.2(2),0.2(2),0.202(2)	−912.0076	−920.4537	−0.92
25	Mn^{16+}	0.04(2),0.19,0.19(2),0.191(2),0.192(2)	−996.2839	−1005.187	−0.89
26	Fe^{17+}	0.039(2),0.181,0.182(2),0.182(2),0.183(2)	−1084.31	−1093.672	−0.86
27	Co^{18+}	0.037(2),0.173,0.174(2),0.174(2),0.175(2)	−1176.087	−1185.906	−0.83
28	Ni^{19+}	0.036(2),0.166,0.166(2),0.167(2),0.168(2)	−1271.614	−1281.892	−0.8

TABLE IX: Neon isoelectronic series. Configuration $1s^2 2s^2 2p^6$. 10 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
9	F^-	0.114(2),0.725(4),0.803(2),5.216(2)	−98.0708	−99.859	−1.79
10	Ne	0.103,0.103,0.656(2),0.664(2),0.667,0.693,0.713,3.781	−126.0531	−128.9383	−2.24
11	Na^+	0.093(2),0.611(8)	−158.3715	−162.0667	−2.28
12	Mg^{2+}	0.085(2),0.53(8)	−194.9627	−199.2214	−2.14
13	Al^{3+}	0.078(2),0.468(8)	−235.555	−240.3925	−2.01
14	Si^{4+}	0.073(2),0.419(8)	−280.1483	−285.5747	−1.9
15	P^{5+}	0.068(2),0.379(8)	−328.7423	−334.7651	−1.8
16	S^{6+}	0.064(2),0.346(8)	−381.337	−387.9615	−1.71
17	Cl^{7+}	0.06(2),0.319(8)	−437.9322	−445.1628	−1.62
18	Ar^{8+}	0.056(2),0.295(8)	−498.528	−506.3678	−1.55
19	K^{9+}	0.053(2),0.275(8)	−563.1243	−571.5758	−1.48
20	Ca^{10+}	0.051(2),0.257(8)	−631.721	−640.7864	−1.41
21	Sc^{11+}	0.048(2),0.242(8)	−704.318	−713.9991	−1.36
22	Ti^{12+}	0.046(2),0.228(8)	−780.9153	−791.2135	−1.3
23	V^{13+}	0.044(2),0.215(8)	−861.5129	−872.4294	−1.25
24	Cr^{14+}	0.042(2),0.204(8)	−946.1108	−957.6465	−1.2
25	Mn^{15+}	0.04(2),0.195(8)	−1034.709	−1046.865	−1.16
26	Fe^{16+}	0.039(2),0.185(8)	−1127.307	−1140.084	−1.12
27	Co^{17+}	0.037(2),0.177(8)	−1223.906	−1237.304	−1.08
28	Ni^{18+}	0.036(2),0.17(8)	−1324.504	−1338.525	−1.05

TABLE X: Sodium isoelectronic series. Configuration $1s^2 2s^2 2p^6 3s^1$. 11 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
11	Na	0.093,0.093,0.61(4),0.612(4),8.927	−158.4273	−162.2554	−2.36
12	Mg^{+}	0.085,0.085,0.528(4),0.532(4),4.43	−195.187	−199.7732	−2.3
13	Al^{2+}	0.078,0.078,0.466(4),0.47(4),2.936	−236.0611	−241.4362	−2.23
14	Si^{3+}	0.073,0.073,0.417(4),0.421(4),2.192	−281.0498	−287.2305	−2.15
15	P^{4+}	0.068,0.068,0.377(4),0.381(4),1.748	−330.1532	−337.1494	−2.08
16	S^{5+}	0.064,0.064,0.344(4),0.348(4),1.453	−383.3708	−391.1892	−2.
17	Cl^{6+}	0.06,0.06,0.317(4),0.321(4),1.243	−440.7025	−449.3474	−1.92
18	Ar^{7+}	0.056,0.056,0.293(4),0.297(4),1.087	−502.1479	−511.6223	−1.85
19	K^{8+}	0.053,0.053,0.273(4),0.276(4),0.965	−567.7067	−578.0129	−1.78
20	Ca^{9+}	0.051,0.051,0.256(4),0.259(4),0.869	−637.3786	−648.5182	−1.72
21	Sc^{10+}	0.048,0.048,0.24(4),0.243(4),0.79	−711.1633	−723.1376	−1.66
22	Ti^{11+}	0.046,0.046,0.227(4),0.229(4),0.724	−789.0608	−801.8706	−1.6
23	V^{12+}	0.044,0.044,0.215(4),0.217(4),0.669	−871.0707	−884.7169	−1.54
24	Cr^{13+}	0.042,0.042,0.204(4),0.206(4),0.621	−957.1929	−971.676	−1.49
25	Mn^{14+}	0.04,0.04,0.194(4),0.196(4),0.58	−1047.427	−1062.748	−1.44
26	Fe^{15+}	0.039,0.039,0.185(4),0.186(4),0.544	−1141.774	−1157.932	−1.4
27	Co^{16+}	0.037,0.037,0.177(4),0.178(4),0.512	−1240.232	−1257.228	−1.35
28	Ni^{17+}	0.036,0.036,0.169(2),0.169,0.17(2),0.17(2),0.171,0.493	−1342.596	−1360.637	−1.33

TABLE XI: Magnesium isoelectronic series. Configuration $1s^2 2s^2 2p^6 3s^2$. 12 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
11	Na^{-}	0.093(2),0.611(8),11.95(2)	−158.4341	−162.275	−2.37
12	Mg	0.085(2),0.53(8),5.101(2)	−195.3048	−200.054	−2.37
13	Al^{+}	0.078(2),0.468(8),3.234(2)	−236.4013	−242.127	−2.36
14	Si^{2+}	0.073(2),0.419(8),2.365(2)	−281.7242	−288.459	−2.33
15	P^{3+}	0.068(2),0.379(8),1.862(2)	−331.2736	−339.036	−2.29
16	S^{4+}	0.064(2),0.346(8),1.535(2)	−385.0495	−393.851	−2.23
17	Cl^{5+}	0.06(2),0.319(8),1.306(2)	−443.0515	−452.899	−2.17
18	Ar^{6+}	0.056,0.056,0.294(2),0.295(4),0.296(2),1.159(2)	−505.1943	−516.178	−2.13
19	K^{7+}	0.053(2),0.274(4),0.275(4),1.025(2)	−571.6248	−583.685	−2.07
20	Ca^{8+}	0.051(2),0.257(8),0.902(2)	−642.4119	−655.42	−1.98
21	Sc^{9+}	0.048(2),0.241(4),0.242(4),0.833(2)	−717.1569	−731.382	−1.94
22	Ti^{10+}	0.046(2),0.228(8),0.748(2)	−796.4441	−811.569	−1.86
23	V^{11+}	0.044(2),0.215(4),0.216(4),0.701(2)	−879.5837	−895.981	−1.83
24	Cr^{12+}	0.042(2),0.205(8),0.639(2)	−967.3737	−984.618	−1.75
25	Mn^{13+}	0.04(2),0.195(8),0.596(2)	−1059.175	−1077.48	−1.7
26	Fe^{14+}	0.039(2),0.186(8),0.558(2)	−1155.199	−1174.565	−1.65
27	Co^{15+}	0.037(2),0.177(8),0.525(2)	−1255.447	−1275.874	−1.6
28	Ni^{16+}	0.036(2),0.17(8),0.496(2)	−1359.919	−1381.407	−1.56

TABLE XII: Aluminium isoelectronic series. Configuration $1s^2 2s^2 2p^6 3s^2 3p^1$. 13 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
13	Al	0.078,0.078,0.467(2),0.468(4),0.468(2),3.655,3.731(2)	−236.5348	−242.347	−2.4
14	Si^{+}	0.073,0.073,0.418(2),0.419(4),0.419(2),2.584,2.639(2)	−282.1046	−289.06	−2.41
15	P^{2+}	0.068,0.068,0.378,0.379,0.379(2),0.379(2),0.38(2),2.002,2.03,2.046	−332.01	−340.145	−2.39
16	S^{3+}	0.064,0.064,0.346,0.346,0.346(2),0.346(2),0.347(2),1.633,1.652,1.667	−386.2511	−395.589	−2.36
17	Cl^{4+}	0.06,0.06,0.318,0.318,0.318(2),0.319(2),0.319(2),1.379,1.392,1.407	−444.8281	−455.384	−2.32
18	Ar^{5+}	0.056,0.056,0.294,0.295,0.295(2),0.295(2),0.296(2),1.193,1.203,1.216	−507.7409	−519.525	−2.27
19	K^{6+}	0.053,0.053,0.274,0.275,0.275(2),0.275(2),0.276(2),1.051,1.058,1.071	−574.9896	−588.01	−2.21
20	Ca^{7+}	0.051,0.051,0.256,0.257,0.257(2),0.257(2),0.258(2),0.939,0.945,0.956	−646.5742	−660.836	−2.16
21	Sc^{8+}	0.048,0.048,0.241,0.241,0.242(2),0.242(2),0.242(2),0.849,0.853,0.864	−722.4945	−738.001	−2.1
22	Ti^{9+}	0.046,0.046,0.227,0.228(2),0.228,0.228(2),0.228(2),0.774,0.778,0.788	−802.7505	−819.504	−2.04
23	V^{10+}	0.044,0.044,0.215,0.215(2),0.216,0.216(2),0.216(2),0.712,0.715,0.724	−887.342	−905.346	−1.99
24	Cr^{11+}	0.042,0.042,0.204,0.204(2),0.205,0.205(2),0.205(2),0.659,0.661,0.67	−976.2688	−995.524	−1.93
25	Mn^{12+}	0.04,0.04,0.194,0.195(2),0.195,0.195(2),0.195(2),0.613,0.615,0.623	−1069.531	−1090.038	−1.88
26	Fe^{13+}	0.039,0.039,0.185,0.185(2),0.186,0.186(2),0.186(2),0.573,0.575,0.582	−1167.128	−1188.888	−1.83
27	Co^{14+}	0.037,0.037,0.177,0.177(2),0.177(2),0.177,0.178(2),0.539,0.54,0.547	−1269.06	−1292.074	−1.78
28	Ni^{15+}	0.036,0.036,0.169,0.17(2),0.17(2),0.17,0.17(2),0.508,0.509,0.515	−1375.327	−1399.594	−1.73

TABLE XIII: Silicon isoelectronic series. Configuration $1s^2 2s^2 2p^6 3s^2 3p^2$. 14 electrons.

Z	Ion	Distances	E_{Bohr}	E_{exact}	Error, %
13	Al^{-}	0.078(2),0.468(8),4.185,4.369,4.372(2)	−236.518	−242.364	−2.41
14	Si	0.073(2),0.418(8),2.857,2.938,2.941(2)	−282.2589	−289.36	−2.45
15	P^{+}	0.068(2),0.379(8),2.16,2.217,2.22(2)	−332.4456	−340.872	−2.47
16	S^{2+}	0.064,0.064,0.346,0.346(2),0.346(2),0.346,0.347(2),1.734,1.779(2),1.783	−387.0779	−396.87	−2.47
17	Cl^{3+}	0.06,0.06,0.318,0.318(2),0.318(2),0.319,0.319(2),1.449,1.486(2),1.49	−446.156	−457.339	−2.45
18	Ar^{4+}	0.056(2),0.295(2),0.295(2),0.295(2),0.295(2),1.26(2),1.276(2)	−509.6792	−522.272	−2.41
19	K^{5+}	0.053(2),0.274(2),0.275(2),0.275(2),0.275(2),1.104(2),1.118(2)	−577.6495	−591.664	−2.37
20	Ca^{6+}	0.051(2),0.257(2),0.257(2),0.257(2),0.257(2),0.982(2),0.994(2)	−650.066	−665.512	−2.32
21	Sc^{7+}	0.048(2),0.241(2),0.242(2),0.242(2),0.242(2),0.884(2),0.895(2)	−726.9289	−743.812	−2.27
22	Ti^{8+}	0.046(2),0.228(2),0.228(2),0.228(2),0.228(2),0.804(2),0.814(2)	−808.2383	−826.565	−2.22
23	V^{9+}	0.044(2),0.215(2),0.216(2),0.216(2),0.216(2),0.737(2),0.746(2)	−893.994	−913.767	−2.16
24	Cr^{10+}	0.042(2),0.204(2),0.205(2),0.205(2),0.205(2),0.68(2),0.688(2)	−984.1961	−1005.419	−2.11
25	Mn^{11+}	0.04(2),0.194(2),0.195(2),0.195(2),0.195(2),0.632(2),0.639(2)	−1078.844	−1101.52	−2.06
26	Fe^{12+}	0.039(2),0.185(2),0.186(2),0.186(2),0.186(2),0.59(2),0.597(2)	−1177.939	−1202.068	−2.01
27	Co^{13+}	0.037(2),0.177(2),0.177(2),0.177(2),0.178(2),0.553(2),0.559(2)	−1281.48	−1307.064	−1.96
28	Ni^{14+}	0.036(2),0.17(2),0.17(2),0.17(2),0.17(2),0.52(2),0.526(2)	−1389.466	−1416.507	−1.91