Bibliography for D-scaling

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(With abstracts)

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equation are constructed. Approximation methods are also discussed within the framework of supersymmetric quantum mechanics and in particular it is shown that a supersymmetry inspired WKB approximation is exact for a class of shape invariant potentials. Supersymmetry ideas give particularly nice results for the tunneling rate in a double well potential and for improving large N expansions. We also discuss the problem of a charged Dirac particle in an external magnetic field and other potentials in terms of supersymmetric quantum mechanics. Finally, we discuss structures more general than supersymmetric quantum mechanics such as para-supersymmetric quantum mechanics in which there is a symmetry between a boson and a para-fermion of order p.

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the Schrodinger equation an explicit recursion relation for the coefficients in the expansion is obtained, and the vanishing of the determinant of these coefficients provides the condition for the energy eigenvalues and for the eigenvectors. The Schrodinger equation is solved and the matrix elements are produced algebraically by using the computer algebra system MAPLE. The substitutions for a particular atom and diagonalization were performed by a program written iu the C language. Since the determinant is sparse, it is possible to go to the order of 1078 as Pekeris did without using excessive memory or computer CPU time. By using a nonlinear variational parameter in the expression used to remove the energy, nonrelativistic energies, within the fixed-nucleus approximation, have been obtained. For the ground-state singlet 1 1S state this is of the accuracy claimed by Frankowski and Pekeris [Phys. Rev. 146, 46 (1966); 150, 366(E) (1966)] using logarithmic terms for Z from 1 to 10, and for the triplet 2(3)S state, energies have been obtained to 12 decimal places of accuracy, which, with the exception of Z = 2, are lower than any previously published, for all Z from 3 to 10.

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lower dimensional theory, as required by general considerations of holography. A subtle cancellation of the entropy coming from the bulk fields in one higher dimension with O(1) contributions from the classical solution ensures that the subleading terms in thermodynamic quantities are of the expected form. While the spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of different spins indicating a need for a nonlocal map between the two sets of fields. We discuss the nature of the propagating degrees of freedom through a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in an Euclidean version are related to nontrivial backgrounds.

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Abstract: The Dirac equation is generalized to D + 1 spacetime case. The radial equations of this quantum system are obtained and solved exactly by the Tricomi equation approach. The energy levels E(n, l, D) are analytically presented. The dependences of the energy E(n, l, D) on the dimension D are also analysed. It is shown that the energy E(n, l, D) (l not equal 0) is almost independent of the quantum number l, while E(n, 0, D) first decreases and then increases as the dimension D increases. Unexpectedly, there is an absence of bound states for this quantum system with the special case D = 1, which is explained in some detail from the fact that the eigenvalues and eigenfunctions do not exist for D = 1.

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Abstract: The Dirac equation is investigated in D + 1 dimensions. The exact Solutions of the Dirac equation with the Coulomb potential are analytically obtained by studying the Tricomi equations.

The eigenvalues and the corresponding fine structure are also discussed.

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D) on the dimension D are analyzed in some detail. The energies E(n, 0, D) first decrease and then increase when increasing dimension D, but the energies E(n, 1, D) (l not equal 0) increase when increasing dimension D. The energies E(n, 0, D) are symmetric with respect to D = 1 for D is an element of (0, 2). It is shown that the energies E(n, l, D) (l not equal 0) are almost independent of the quantum number I for large D and are completely independent of it if the Coulomb potential is equal to the scalar one. The energies E(n, l, D) almost overlap for large D. The dependences of the energies E(n, l, v) and E(n, l, s) on the vector potential parameter v and scalar potential one s are also studied for D 3. All are found to decrease when these parameters are increased. (C) 2004 Wiley Periodicals, Inc.

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for series summation to provide a powerful tool well suited to the numerical analysis of bifurcations in nonlinear problems. Numerical results are presented for a variety of applications, including examples from fluid mechanics.

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lowest energy level of the P-state with spin zero is higher than that of the state with spin one if the parity and the spatial dimension D are the same, and that I E I of the states with the same angular momentum and the parity decreases as D increases. Moreover, the interdimensional degeneracies are confirmed in the calculation, namely the states P-1,3(e) and D-1,3(o) in three dimensions are degenerated with the states S-3,1(e) and P-3,1(o) in five dimensions. (C) 2004 Elsevier B.V All rights reserved.

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Abstract: Rost et al. have examined some of the implications of dimensional scaling for the molecular orbital (MO) description of the "right parity" (pi = (-1)L) doubly-excited states of two-electron atoms. They propose a new and simple classification scheme utilizing interdimensional degeneracies, saddle degeneracies, and A-doubling. The present paper extends this description, in D dimensions, to the "wrong parity" (pi = (-1)L+1) states and provides a justification for the extension of the new classification scheme to include the pi = (-1)L+1 states. This is achieved by showing that

the MO approximation preserves the exact interdimensional degeneracies between pi = (-1)L and pi = (-1)L+1 states and that there exists approximate interdimensional degeneracies between the pi = (-1)L+1 MO states. It is also pointed out that the phenomenon of LAMBDA-doubling is special to three dimensions, although the lifting of this degeneracy is expected to be small for low dimensions. Despite the fact that the energy level spectrum no longer features LAMBDA-doubling for larger values of D, the internal MO electron center of mass wavefunctions for those states which exhibit LAMBDA-degeneracy at D = 3 are still identical in any dimension. The lifting of this degeneracy does not affect the classification scheme of Rost et al. since it utilizes LAMBDA-doubling at D = 3.

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recently and is an N-electron D-dimensional generalization of the Schwartz expansion. A derivation of the coupled differential equations in the internal variables that result from the application of the Hamiltonian to this wave-function expansion for the atomic two-electron system has been presented by the authors in another recent paper. The coupled differential equations admit continuation in D and clearly show the complete spectrum of exact interdimensional degeneracies of the two-electron system. However, to apply the methods of dimensional scaling to the two-electron system, the system of coupled differential equations have to be solved for large D. This paper concerns itself with this issue. [S1050-2947(97)07608-7].

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but solvable, potentials suggests that shape invariance is not only sufficient but perhaps even necessary for the SWKB approximation to be exact. A comparison of the WKB and SWKB predictions for the bound-state spectra of a number of potentials reveals that in many cases the SWKB approach does better than the usual WKB approximation.

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is applied and subsequent extrapolations gives the continuous grid result. The program is written in C and is highly optimized. All computations are made in double precision. Despite this relatively low degree of floating point precision (48 digits are not uncommon), the accuracy in the results can reach about 10 significant figures. Both serial and parallel versions are provided. The parallel program is particularly suitable for shared memory machines such as the Sun Starcat series. The serial version is simple to compile and should run on any platform.

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with modifications that that completely remove the singularities at delta = 1 and remove the dominant singularity at delta = 0 are demonstrated. A renormalization of the interelectron repulsion is found to move the dominant singularity of the Borel function F(delta) = Sigma(j)E(j)'/ j!, where E-j' are the the expansion coefficients of the energy with singularity structure removed at d51, farther from the origin and thereby accelerate summation convergence. The ground-state energies of He and H-2(+) are used as test cases. The new methods give significant improvement over previous summation methods. Shifted Borel summation using F-m(delta) = Sigma(j)E(j)'/Gamma(j + 1 - m) is considered. The standard deviation of results calculated with different values of the shift parameter m is proposed as a measure of summation accuracy. (C) 1998 American Institute of Physics. [S0022-2488(98)04210-8].

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Abstract: The large-N limit of the hermitian matrix model in three and four euclidean space-time dimensions is studied with the help of the approximate Renormalization Group recursion formula. The planar graphs contributing to wave-function, mass and coupling-constant renormalization are

identified and summed in this approximation. In four dimensions the model fails to have an interacting continuum limit, but in three dimensions there is a non-trivial fixed point for the approximate RG relations. The critical exponents of the three-dimensional model at this fixed point are nu = 0.67 and eta = 0.20. The existence (or non-existence) of the fixed point and the critical exponents display a fairly high degree of universality since they do not seem to depend on the specific (non-universal) assumptions made in the approximation.

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Abstract: We present a detailed study of the ground state behaviour of two-electron diatomic molecules. The ground state stability diagram for diatomic molecules in the Born-Oppenheimer approximation is obtained and the behaviour of the ground state near the stability line is studied. Two different cases are analysed: the homonuclear two-centre two-electron molecule with the internuclear distance as a free parameter and the diatomic two-electron molecule (in this case, the internuclear distance is determined by equilibrium conditions). Analytical and numerical results for these systems are presented.

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Abstract: We present dimensional scaling calculations for the critical parameters needed to bind one and two-electrons to a finite linear dipole field and the stability diagram for the hydrogen-antihydrogen like molecules. We find that calculations at the large-D limit are much simpler that D = 3, yet yield similar results for the critical parameters and the stability diagrams. (C) 2008 Elsevier B. V. All rights reserved.

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Abstract: We present finite size scaling calculations of the critical parameters for binding two electrons to a finite linear dipole field. This approach gives very accurate results for the critical parameters by using a systematic expansion in a finite basis set. A complete ground state stability diagram for the dipole-bound dianion is obtained using accurate variational and finite size scaling calculations. We also study the near threshold behavior of the ground state energy by calculating its critical exponent.

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Abstract: We present a detailed study of the stability conditions of hydrogen-antihydrogen-like quasimolecules using both variational and finite-size scaling calculations. The stability diagram of the nuclear charge Z as a function of the intemuclear distance R shows bound and unbound regions separated by a first-order critical line. Calculations of the leptonic annihilation rate show a peculiar behavior for nuclear charges Z ¿= 2, which was not observed for the hydrogen-antihydrogen quasimolecule; it goes through a maximum before it decays exponentially for large interhadronic distances. This might have a practical impact on the study of stability of matter-antimatter systems.

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Abstract: A Rydberg atom embedded in a plasma can experience penetration by slowly moving electrons within its volume. The original pure Coulomb potential must now be replaced by a screened Coulomb potential which contains either a screening length R-s or a screening factor A = R-s(-1). For any given discrete energy level, there is a Critical Screening Factor (CSF) A(c) beyond which the energy level disappears (by merging into the continuum). Analytical results are obtained for the classical dependence of the energy on the screening factor, for the CSF, and for the critical radius

of the electron orbit for Circular Rydberg States (CRS) in this screened Rydberg atom. The results are derived for any general form of the screened Coulomb potential and are applied to the particular case of the Debye potential. We also show that CRS can temporarily exist above the ionization threshold and are therefore the classical counterparts of quantal discrete states embedded into continuum. The results are significant not only to Rydberg plasmas, but also to fusion plasmas, where Rydberg states of multi-charged hydrogen-like ions result from charge exchange with hydrogen or deuterium atoms, as well as to dusty/complex plasmas.

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Abstract: Witten's generalization to arbitrary dimension has afforded new insight into the correlated motion of quantum particles [Phys. Today 33, (7), 38 (1980)]. We have used a classically based method to understand the resultant dimensionality dependence of the ground-state energy of the helium atom in the approximation which regards the quantum fluctuations of the system as being harmonic oscillations about a classical, correlated state of minimum effective potential energy. Making an analogy with thermal systems, this provides a "phase diagram" of a single helium atom that features a first-order melting transition, with inverse dimensionality playing the role of temperature. Our approximation gives an understanding of the high-dimensionality behavior of the quantum solution found with a perturbation theory expansion in inverse dimensionality by Goodson and Herschbach [Phys. Rev. Lett. 58, 1628 (1987)]. From comparison with variational quantum ground-state solutions by Loeser and Herschbach [J. Chem. Phys. 84, 3882 (1986)] for atomic numbers 2, 3, and 6 we find that the harmonic description improves with decreasing nuclear charge.

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> **Abstract:** Directional superpositions of the rotational states of linear molecules can:be created by interaction of the polarizability with the strong electric field of an intense laser beam. The polarizability interaction, proportional to cos(2) theta (with theta the angle between the molecular axis and the direction of the electric field), is a double-well potential, with end-for-end symmetry. The energy levels thus exhibit tunnel-effect splittings. We present an exact treatment, derived by transcription to an oblate spheroidal wave equation. We also develop an accurate semiclassical approximation, in which the action integral and period are evaluated using an effective potential that corresponds to one-dimensional motion. The results pertain both to rotational spectroscopy of nonpolar molecules and to alignment and trapping experiments with either nonpolar or polar molecules.

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Abstract: We extend and apply a previously developed method for a semiclassical treatment of a system with large spin S. A multisite Heisenberg Hamiltonian is transformed into an effective classical Hamilton function which can be treated by standard methods for classical systems. Quantum effects enter in form of multispin interactions in the Hamilton function. The latter is written in the form of an expansion in powers of J/(TS), where J is the coupling constant. Main ingredients of our method are spin coherent states and cumulants. Rules and diagrams are derived for computing cumulants of groups of operators entering the Hamiltonian. The theory is illustrated by calculating the quantum corrections to the free energy of a Heisenberg chain which were previously computed by a Wigner-Kirkwood expansion.

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Abstract: By performing first-order perturbation theory about the m –¿ infinity limit, we obtain a highly accurate analytical expression for circular Rydberg (m = n - 1 much greater than 1) wavefunctions of the hydrogen atom in a uniform magnetic

field. In the m –¿ infinity limit, the electron executes a circular orbit about the magnetic field axis; the first-order wavefunction simply corresponds to Gaussian fluctuations about this orbit. To demonstrate the utility of the results derived here, we compare our simple expression for the lifetimes of m = 25 and 35 circular states with accurate ab initio results.

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Abstract: Dimensional perturbation theory is applied to the calculation of complex energies for quasibound (resonance) eigenstates, using a modified dimension-dependent potential so that the infinite-dimensional limit better reflects the physical (three-dimensional) nature of the resonant eigenstate. Using the previous approach of retaining the D=3 form of the potential for all spatial dimension D, highly accurate results are obtained via Pade-Borel summation of the expansion coefficients when they are complex, but a lesser degree of convergence is found when quadratic Pade summation is applied to real expansion coefficients. The present technique of using a dimension-dependent potential allows complex expansion coefficients to be obtained in all cases, and is demonstrated to provide a marked improvement in convergence. We illustrate this approach on the Lennard-Jones potential and the hydrogen atom in an electric field. (C) 1996 American Institute of Physics.

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Abstract: Dimensional pertubation theory is applied to the calculation of complex energies for quasibound, or resonant, eigenstates of central potentials. Energy coefficients for an asymptotic expansion in powers of 1/kappa, where kappa = D +

2l and D is the Cartesian dimensionality of space, are computed using an iterative matrix based procedure. For effective potentials which contain a minimum along the real axis in the kappa –¿ infinity limit, Hermite-Pade summation is employed to obtain complex eigenenergies from real expansion coefficients. For repulsive potentials, we simply allow the radial coordinate to become complex and obtain complex expansion coefficients. Results for ground and excited states are presented for squelched harmonic oscillator (V0r2e(-r) and Lennard-Jones (12-6) potentials. Bound and quasibound rovibrational states for the hydrogen molecule are calculated from an analytic potential. We also describe the calculation of resonances for the hydrogen atom Stark effect by using the separated equations in parabolic coordinates. The methods used here should be readily extendable to systems with multiple degrees of freedom.

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Abstract: We apply dimensional perturbation theory to the calculation of Regge pole positions, providing a systematic improvement to earlier analytic first-order results. We consider the orbital angular momentum l as a function of spatial dimension D for a given energy E, and expand l in inverse powers of kappa=(D-1)/2. It is demonstrated for both bound and resonance states that the resulting perturbation series often converges quite rapidly, so that accurate quantum results can be obtained via simple analytic expressions given here through third order. For the quartic oscillator potential, the rapid convergence of the present l(D;E) series is in marked contrast with the divergence of the more traditional E(D;l) dimensional perturbation series, thus offering an attractive alternative for bound state problems. (C) 1997 American Institute of Physics.

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Abstract: We describe the development of Lewis's ideas concerning the chemical bond and in particular the concept of the electron pair bond and the octet rule. We show that the concept of the electron pair bond has endured to the present day and is now understood to be a consequence of the Pauli principle. In contrast the octet rule is now regarded as much less important than was originally generally believed, although Lewis himself knew several exceptions and regarded it as less important than what he called the rule of two (the electron pair). The octet rule was more strongly promoted by Langmuir who is also responsible for the term covalent bond. However, many more exceptions to the octet rules than were known to Lewis are now known and the terms hypervalent and hypovalent used to describe such molecules are no longer particularly useful. Today it is realized that bonding electron pairs in many molecules are not as well localized as Lewis believed, nevertheless resonance structures, i.e., plausible alternative Lewis structures, are still often used to describe such molecules. Moreover electrons are not always found in pairs, as for example in linear molecules, which can, however, be satisfactorily described by Linnett's double quartet theory. The electron density distribution in a molecule can now be analyzed using the ELF and other functions of the electron density to show where electron pairs are most probably to be found in a molecule. (C) 2006 Wiley Periodicals, Inc.

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Abstract: In this paper we give a closed form expression for the 1/d corrections to the leading d

–¿ infinity approximation for the ground-state energy of an n-body quantum mechanical problem in a general potential (where d is the number of spatial dimensions). We then extend the results to the problem of directed polymers in a random medium where the limit n –¿ 0 has to be taken in the corresponding n-body problem. In this case the effective two-body potential measures the strength of correlations of the quenched disorder. In the n –¿ 0 limit a replica-symmetry-breaking solution to the saddle-point equations turns out to be the correct solution in the strong-coupling regime, and we obtain the 1/d corrections to the corresponding energy which constitute the gaussian fluctuations about this solution as well as provide information about its basin of stability.

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Abstract: Qualitative properties of the ground states and the low-lying spectra of three- and four-body systems are studied in the framework of the 1/D method, where D is the dimensionality of space. Special emphasis is laid on model-independent properties and relations between magnitudes. It is shown that the low terms of the 1/D expansion often give a good qualitative description of the system, although the 1/D series is known to

exhibit poor convergence for D = 3. The Post estimate of the ground-state energy, the ordering of Wigner multiplets in the spectrum of resonances of the He-4 nucleus, the Hund rule in the two-electron atom, and other properties follow straightforwardly from the 1/D analysis.

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Abstract: We present simple estimates of the binding energies of clusters of rare-gas atoms and excitation energies of low-lying states in trimers, which are obtained by formally writing the Schrodinger equation in d dimensions and expanding its solution in powers of 1/d. Comparison with Monte Carlo calculations for Lennard-Jones clusters (Ne(n), Ar(n), Kr(n), Xe(n), n = 3-7 and 13) shows good agreement, and suggests that our formulae may be used to estimate the energy of rare-gas atoms interacting through realistic potentials.

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Abstract: Within the framework of the nonrelativistic quark model and with the help of the leading term of the 1/d-expansion we obtain analytical expressions for the quotients of baryon-to-meson strong interaction radii. Phenomenological relativistic corrections a la Povh and Hufner are shown to lower the non relativistic estimate. Predictions for baryon radii are given and a comparison with experimental results (when available) is made. For some values of the quark masses, our estimate differs from the naive parton result in more than 20%.

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Abstract: A method is developed for using large-order perturbation theory to solve the systems of coupled differential equations that result from the

variational solution of the Schrodinger equation with wave functions of product form. This is a noniterative, computationally efficient way to solve self-consistent-field (SCF) equations. Possible applications include electronic structure calculations using products of functions of collective coordinates that include electron correlation, vibrational SCF calculations for coupled anharmonic oscillators with selective coupling of normal modes, and ab initio calculations of molecular vibration spectra without the Born-Oppenheimer approximation.

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Abstract: Rational and algebraic Pade approximants are applied to Moller-Plesset (MP) perturbation expansions of energies for a representative sample of atoms and small molecules. These approximants can converge to the full configuration-interaction result even when partial summation diverges. At order MP2 (the first order beyond the Hartree-Fock approximation), the best results are obtained from the rational [0/1] Pade approximant of the total energy. At MP3 rational and quadratic approximants are about equally good, and better than partial summation. At MP4, MP5, and MP6, quadratic approximants appear to be the most dependable method. The success of the quadratic approximants is attributed to their ability to model the singularity structure in the complex plane of the perturbation parameter. Two classes of systems are distinguished according to whether the dominant singularity is in the positive half plane (class A) or the negative half plane (class B). A new kind of quadratic approximant, with a constraint on one of its constituent polynomials, gives better results than conventional approximants for class B systems at MP4, MP5, and MP6. For CH3 with the C-H distance at twice the equilibrium value the quadratic approximants yield a complex value for the ground-state electronic energy. This is interpreted as a resonance eigenvalue embedded in the ionization continuum. (C) 2000 American Institute of Physics. [S0021-9606(00)30208-2].

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> **Abstract:** The accuracy of Moller-Plesset (MP) perturbation theory and coupled-cluster (CC) theory can be significantly improved, at essentially no increase in computational cost, by using summation approximants that model the way in which these theories converge to the full configuration interaction limit. Approximants for MP4 and CCSD(T) are presented, their size scaling is analyzed, and the functional analysis of the MP energy, on which the MP4 approximant is based, is discussed. The MP approximants are shown to have a form that is appropriate for describing resonance energies. (C) 2003 Wiley Periodicals, Inc.

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> **Abstract:** We examine strategies for approximating the sum of a perturbation expansion for Coulombic systems in inverse powers of the spatial dimension D using only the few lowest-order terms. Several summation methods are tested on energy expansions of the following systems: the ground state of He, the ground state of Li+, the first two excited S states and the lowest 3P(e) state of He, and the ground state of H-2+. The origin of the expansions, the limit D –¿ infinity, corresponds to a pseudoclassical limit, but lower-order terms are strongly affected by poles at D –¿ 1, a hyperquantum limit. Two alternative methods are recommended, weighted truncation and hybrid Pade summation, which exploit these poles to reduce the summation error. In effect, these methods modify a

semiclassical 1/D expansion by incorporating corrections that are often large. Weighted truncation appears to be slightly more dependable at very low orders, while hybrid Pade summation is preferable when more than the three lowest-order terms are available or when the residue of the second-order pole at D = 1 is known. We demonstrate that the present methods are superior to the shifted-expansion method, which does not correctly model the singularity structure.

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Abstract: An asymptotic expansion for the electronic energy of two-electron atoms is developed in powers of delta=1/D, the reciprocal of the Cartesian dimensionality of space. The expansion coefficients are calculated to high order (approximately 20 to 30) by an efficient recursive procedure. Analysis of the coefficients elucidates the singularity structure in the D –¿ infinity limit, which exhibits aspects of both an essential singularity and a square-root branch point. Pade-Borel summation incorporating results of the singularity analysis yields highly accurate energies; the quality improves substantially with increase in either D or the nuclear charge Z. For He, we obtain 9 significant figures for the ground state and 11 for the 2p2 3P(e) doubly excited state, which is isomorphic with the ground state at D=5 by virtue of interdimensional degeneracy. The maximum accuracy obtainable appears to be limited only by accumulation of roundoff error in the expansion coefficients. The method invites application to systems with many electrons or subject to external fields.

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Abstract: For a nonrelativistic hydrogenic atom, the dimension dependence of the energy levels is non-singular except for a second-order pole at D*

= 3 - 2n, where n is the principal quantum number. For the relativistic Klein-Gordon and Dirac equations, the dimension dependence has a much more complicated singularity structure, involving branch points. For all eigenstates there are branch points at D+ +/- 2Z-alpha, where Z is the nuclear charge, alpha is the fine-structure constant, and D+ is independent of n but varies linearly with orbital angular momentum. For most states there is in addition a pair of branch points near D* but slightly off the real axis. The customary perturbation expansion in terms of Z-alpha gives qualitatively incorrect dimension dependence; it predicts only poles located on the real axis at D* and D+, no matter how high the order of the expansion. The dimensional singularities result from the behavior at r = 0. The qualitatively incorrect results occur because the perturbing potential, proportional to alpha-2/r2, overwhelms the unperturbed 1/r potential at small r. Because of the complexity of the dimensional singularity structure, the popular "shifted expansion" method for summing the 1/D expansion does not work well for these equations. We demonstrate a general method for identifying the dimensional singularities that leads to an exact summation of the 1/D expansion for all eigenstates of both the Klein-Gordon and the Dirac equations for a particle in a Coulomb potential.

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Abstract: The usefulness of Moller-Plesset perturbation theory, a standard technique of quantum chemistry, is determined by singularities in the corresponding energy function in the complex plane

of the perturbation parameter. A method is developed that locates singularities from fourth-order perturbation series, using quadratic approximants with bilinear conformal mappings. (c) 2006 Elsevier B.V. All rights reserved.

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Abstract: Large-order dimensional perturbation theory, which yields high accuracy for ground-state energies, is applied here to excited states of the two-electron atom. Expansion coefficients are computed recursively using the moment method, which we formulate in terms of normal coordinates. We consider the first two excited S states of helium, corresponding, at the large-dimension limit, to one quantum in either the antisymmetric-stretch normal mode or the symmetric-stretch normal mode. Comparison with the hydrogenic limit has identified these states as Is 2s 3S and 1s2s 1S, respectively. We sum the 1/D expansions at D = 3, using summation procedures that take into account the dimensional singularity structure of the eigenvalues, and find convergence at D = 3 to the eigenvalues predicted by the hydrogenic assignments, despite apparent qualitative differences between the eigenfunctions at large D and those at D = 3. In the D –¿ infinity limit, the electrons are equidistant from the nucleus. Our results for 1s2s energies appear to imply that the shell structure is properly accounted for by terms in the expansion beyond the lowest order. This robustness of the 1/D expansion suggests that the method will be applicable to many-electron systems.

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Abstract: For a two-electron atom, many D = 3 states have the same energies as D = 5 states of lower angular momentum. Thus the energies of 3P(e), 1P(e), 3D0, and 1D0 states for D = 3 are respectively identical to those for 1S(e), 3S(e), 1P0,

and 3P0 states at D = 5. We exploit these interdimensional degeneracies to obtain accurate energies for doubly excited 2pnp P(e) states of helium at D = 3, with n = 2-6, by calculating energy eigenvalues for the singly excited 1s(n -1)s S(e) states at D = 5. We also illustrate how some qualitative aspects of double-excitation spectra can be elucidated in terms of interdimensional degeneracies.

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Abstract: The integral equations of Gelfand and Levitan were previously used by Abraham and Moses (Phys. Rev. A, 22 (1980) 1333) to construct a modified local potential V-pert(r), which in this work is added to a coulombic potential in order to remove the lowest s state from the spectrum. The total modified potential V-new(r), for excited states is used in conjunction with the shifted 1/D expansions of dimensional scaling. It is verified that it is feasible to find the lowest energies and the mean radius for V-new, treating it now as a ground state problem. This indicates that a potential for excited states can be generated by the procedure, with the possibility of using it in the study of excited states and valence states. The potentials applied can be considered as local pseudopotentials.

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Abstract: The independent eigenstates of the total orbital angular momentum operators for a three-body system in an arbitrary D-dimensional space are presented by the method of group theory. The Schrodinger equation is reduced to the generalized radial equations satisfied by the generalized radial functions with a given total orbital angular momentum denoted by a Young diagram [mu, nu, 0,..., 0] for the SO(D) group. Only three internal variables are involved in the functions and equations. The number of both the functions and the equations for the given angular momentum is finite and equal to (mu-nu+1). (C) 2002 American Institute of Physics.

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Abstract: The Dirac equation is generalized to D + 1 space-time. The conserved angular momentum operators and their quantum numbers are discussed. The eigenfunctions of the total angular momenta axe calculated for both odd D and even D cases. The radial equations for a spherically symmetric system are derived. The exact solutions for the system with a Coulomb potential axe obtained analytically. The energy levels and the corresponding fine structure axe also presented.

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Abstract: A new approach is developed to derive the complete spectrum of exact interdimensional degeneracies for a quantum three-body system in D dimensions. The new method gives a generalization of previous methods. (C) 2002 Elsevier Science B.V. All rights reserved.

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Abstract: We study the properties of an isolated, self-interacting wormlike polymer chain on the basis of a nonperturbative 1/d-expansion, where d denotes the dimension of embedding space. In the absence of an external force, we characterize the dimension R of the chain in embedding space via R similar toL(nu), where L is the internal size. (A) Long-range, repulsive segmental interactions decaying as 1/r(alpha) may control chain conformations that are either rodlike, nu =1(1 ¡ alpha ¡2), "wrinkled," 1/2 ¡ nu ¡1(2 ¡ alpha ¡4), or random-walk-like, nu =1/2(alpha ¿4). (B) For short-range, screened, repulsive interactions, the crossover between rodlike and random-walk-like behavior is controlled by the persistence length whose interaction part we compute focusing on a Debye-Huckel interaction of strength V-0, with inverse screening length kappa (0). The induced persistence length varies as V(0)(beta)kappa (-gamma)(0), with, as expected, (beta,gamma)=(1,2) when the chain is intrinsically stiff, and, surprisingly, with either (beta,gamma)=(1/6,7/6) or (beta,gamma)=(1,7) when the chain is intrinsically very flexible. The chances of experimentally observing the novel regimes may be limited. For a chain subject to an external stretching force f, we determine the force-extension relation zeta=zeta (f)=zeta (0)+delta zeta (f), where zeta denotes the chain extension, zeta (0) is the spontaneous extension. (A) If the interaction potential is either screened, or if the decay of a long-range interaction potential is fast, i.e., if alpha ¿4, the chain spontaneously generates an "effective tension" and responds linearly to weak forces with elastic constants "renormalized" by interactions. By contrast, "tension-free" chains, with either nu =1, where delta zeta similar tof(1/2), or with nu =2/alpha, where delta zeta similar tof(1/3), respond to the weakest force nonlinearly. (B) Near full extension the chain always responds nonlinearly. When the potential is screened, or if alpha ¿4, we find the 1/rootf corrections typical of wormlike chains. (C) 2001 American Institute of Physics.

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Abstract: One type of Bohr orbit approach to descriptions of the electronic structures of atoms and molecules is based on a 2n x n factorization of the atomic shell-structure formula 2n(2). It involves n circular orbits with principal quantum number n, each of which may accommodate a maximum of 2n electrons. Aspects of a previously published account of this type of orbit theory are reviewed for compounds of main-group elements, and for several transition metal complexes. New types of orbit diagrams are provided for some of the systems considered previously. The theory provides a straightforward representation for the tetrahedral carbon atom. Attention is given to orbit diagrams for a variety of hypercoordinate systems, and to the general reluctance of atoms of first-row elements to form stable hypercoordinate compounds. Orbit diagrams for two elementary triatomic reactions are also provided, and examples are provided of new types of Bohr orbit calculations of the energies for H-2(+) and H-2.

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Abstract: Examples are provided of Bohr circular orbit diagrams to represent the electronic structures of some fluorine-containing molecules. The orbit 2 diagrams are constructed from a 2n x n factorisation of the atomic shell-structure formula 2n, with n = 1, 2, 3, Particular attention is given to orbit diagrams and the associated valence bond structures for the hypercoordinate molecules and ions PF5 and NF5, F-3(-) and XeF2, IF5 and XeF5+, XeF5-, IF8-, XeFg(8)(2-), ReF8- and TaF83-, ZrF84-, ZrF73-, ReF62-, and high-spin CoF63-. Aspects of the electronic structures of D-3h-symmetry PF5 and NF5 are contrasted via the

use of orbital valence bond considerations, and the results of STO-3G valence bond calculations are reported for these species. (c) 2006 Elsevier B.V. All rights reserved.

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the hyperradius. This enables one both to reproduce a very precise value of the ground state energy and to obtain much better agreement of the theory with the experiment for higher energies of 1sns-configurations for singlet as well as for triplet states.

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Abstract: A nostalgic account is given of my scientific odyssey, recalling early encounters, some fateful, some just fun, with mentors, methods, and molecules. These include stories of my student years at Stanford, pursuing chemical kinetics with Harold Johnston; graduate study at Harvard, doing molecular spectroscopy with Bright Wilson; and fledgling faculty years at Berkeley, launching molecular beam studies of reaction dynamics. A few vignettes from my "ever after" era on the Harvard faculty emphasize thematic motivations or methods inviting further exploration. An Appendix provides a concise listing of colleagues in research and the topics we have pursued.

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Abstract: Chief features of dimensional scaling methods are exemplified by briefly reviewing prototypical applications and recent developments. The pseudoclassical large-D limit usually can be evaluated exactly regardless of the magnitude, nature, and number of strong, nonseparable dynamical interactions. Often, this limit can be accurately Linked to D = 3 by perturbation or interpolation methods. This is because the dimension dependence of many-body effects tends to be smooth and mild when calibrated by appropriate one- or few-body problems. A simple renormalization procedure applied to atoms with up N similar to 100 electrons yields a major part of the correlation energy. From Hartree-Fock input, a renormalized nuclear charge is determined which renders the dimensionally scaled energy at D –¿ infinity a good approximation to that for D = 3 with the actual Z. Prospects are discussed for other means to exploit dimensional scaling, including an analogous renormalization procedure for molecules. (C) 1996 John Wiley & Sons, Inc.

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Abstract: For arbitrary level filling, the fully nonlocal kinetic energy density and Pauli potential for the one-dimensional harmonic oscillator can be constructed explicitly. In the present work, these exact results are eventually compared with the low-order gradient expansions. This prompts a fuller study of the dimensionality dependence of low-order gradient expansions for systems with general one-body potentials, and its relevance to the theory of the Pauli potential. One consequence of the present work is to display the generalization to D-dimensions as (D-2)/3D of the three-dimensional Kirzhnits coefficient 1/9 of the von Weizsacker term in the kinetic energy density.

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Abstract: General and recursive formulae for the degeneracy of the angular momentum I of the atomic orbitals of an n-dimensional atom are derived from the algebraic manipulation of homogeneous polynomials. General rectangular coordinate expressions for the wavefunctions of the p- and d-orbitals of an n-dimensional atom are also obtained. The analytical expressions for the sixteen f-orbitals of a four-dimensional atom are given. The relation between the wavefunctions of these hypothetical atoms and those for the three-dimensional atoms and molecules is discussed.

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Abstract: By taking a birds-eye view from the n-dimensional world (or n-space), it was found that the conventionally used sp, sp(2), and sp(3) hybridized atomic orbitals belong to the sp(n) hybridization and their geometrical shapes correspond to n-simplexes, which are, respectively, the smallest geometrical objects in n-space. Similarly, sp, sp(2)d, and sp(2)d(3) hybridizations are found

to belong to the sp(n)d(n-1) hybridization, whose geometrical shapes correspond to n-cross polytopes (obtained from 2n vertices equidistantly located on n rectangular coordinate axes). Another series of n-cube hybridization is also discussed, whose 3-space member is 8-coordinated cubic hybridization, sp(3)d(3)f. General analytical forms of the wave-functions of these three series of hybridized atomic orbitals in n-space are obtained. The periodic table and related problems of atoms and molecules in n-space are discussed.

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Abstract: A renormalization of the D-dimensional Hamiltonian is developed to ensure that the large-D limit corresponds to a single well at any value of the internuclear distance R. This avoids convergence problems caused by a symmetry-breaking transition that is otherwise expected to occur when R is approximately equal to the equilibrium bond distance R-eq, With larger R giving a double well. This symmetry breaking has restricted the applicability of large-order perturbation theory in 1/D to cases where R is significantly less than R-eq. The renormalization greatly extends the range of R for which the large-order expansion can be summed. A numerical demonstration is presented for H-2(+). The 1/D expansions are summed using Pade-Borel approximants with modifications that explicitly model known singularity structure.

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Abstract: We present the exact energy bound state solutions of the Schrodinger equation in D-dimensions for an alternative (often used) pseudo-Coulomb potential plus ring-shaped potential of the form V (r) = -a/r+b/r(2)+beta cos(2)theta/r2sin(2)theta+c. The solution was obtained analytically by means of the Nikiforov-Uvarov method. We give a clear recipe of how to

obtain an explicit solution to the D-dimensional radial and angular parts of the wave functions in terms of orthogonal polynomials. The total energy of the system is different from the pseudo-Coulomb potential because of the contribution of the angular part. The general results obtained in this work can be reduced to the well-known three-dimensional forms given in the literature.

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Abstract: We present a new approximation scheme for the centrifugal term to solve the Schrodinger equation with the Hulthen potential for any arbitrary l-state by means of a mathematical Nikiforov-Uvarov (NU) method. We obtain the bound-state energy eigenvalues and the normalized corresponding eigenfunctions expressed in terms of the Jacobi polynomials or hypergeometric functions for a particle exposed to this potential field. Our numerical results of the energy eigenvalues are found to be in high agreement with those results obtained by using the program based on a numerical integration procedure. The s-wave (l = 0) analytic solution for the binding energies and eigenfunctions of a particle are also calculated. The physical meaning of the approximate analytical solution is discussed. The present approximation scheme is systematic and accurate.

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Abstract: The energy eigenvalues of bound states of an electron in the general exponential cosine screened Coulomb potential are obtained using the shifted 1/N expansion method. The energies for the states from 1s to 8k are calculated from six to eight significant figures. The energy eigenvalues for the 1s, 2s - 2p, 3s - 3d, and 4s - 4f states are also presented as a function of the screening parameter lambda. Results are compared with the ones obtained by other workers. The agreement reduces

roughly for large lambda. It is also observed that the convergence of the expansion series increases remarkably as l increases.

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Abstract: In the framework of potential models for heavy quarkonium, we compute the mass spectrum of the bottom-charmed B-c meson system and spin-dependent splittings from the Schrodinger equation using the shifted-large-N expansion technique. The masses ;of the lightest vector B-c(*) and pseudoscalar B-c states as well as the higher states below the threshold are estimated. Our predicted result for the ground state energy is 6253(-6)(+15) MeV and are generally in exact agreement with earlier calculations. Calculations of the Schrodinger energy eigenvalues are carried out up to the third order of the energy series. The parameters of each potential are adjusted to obtain best agreement with the experimental spin-averaged data (SAD). Our findings are compared with the observed data and with the numerical results obtained by other numerical methods.

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Abstract: A new approximation formalism is applied to study the bound states of the Hellmann potential, which represents the superposition of the attractive Coulomb potential -a/r and the Yukawa potential bexp(-delta r)/r of arbitrary strength h and screening parameter delta. Although the analytic expressions for the energy eigenvalues E-n,E-l yield quite accurate results for a wide range of n, f in the limit of very weak screening, the results become gradually worse as the strength b and the screening coefficient 6 increase. This is because that the expansion parameter is not sufficiently small enough to guarantee the convergence of the expansion series for the energy levels. (C) 2007 Elsevier B.V. All rights reserved.

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Abstract: An alternative approximation scheme has been used in solving the Schrodinger equation for the exponential-cosine-screened Coulomb potential. The bound state energies for various eigenstates and the corresponding wave functions are obtained analytically up to the second perturbation term.

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Abstract: An alternative approximation scheme has been used in solving the Schrodinger equation to the more general case of exponential screened Coulomb potential, V(r) = -(a/r)[1 + (1 + br)e(-2br)]. The bound state energies of the 1s, 2s and 3s-states, together with the ground state wave function are obtained analytically upto the second perturbation term.

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Abstract: By using an ansatz for the eigenfunction, we have obtained the exact analytical solutions of the radial Schrodinger equation for the

pseudoharmonic and the Kratzer potentials in two dimensions. The bound-state solutions are easily calculated from this eigenfunction ansatz. The corresponding normalized wavefunctions are also obtained. (C) Versita Warsaw and Springer-Verlag Berlin Heidelberg. All rights reserved.

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Abstract: The polynomial solution of the D-dimensional Schrodinger equation for a special case of Mie potential is obtained with an arbitrary l not equal 0 states. The exact bound state energies and their corresponding wave functions are calculated. The bound state (real) and positive (imaginary) cases are also investigated. In addition, we have simply obtained the results from the solution of the Coulomb potential by an appropriate transformation. (C) 2008 Elsevier B.V. All rights reserved.

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Abstract: The Schrodinger equation in D-dimensions for the Manning-Rosen potential with the centrifugal term is solved approximately to obtain bound states eigensolutions (eigenvalues and eigenfunctions). The Nikiforov-Uvarov (NU) method is used in the calculations. We present numerical calculations of energy eigenvalues to two- and four-dimensional systems for arbitrary quantum numbers n and 1, with three different values of the potential parameter alpha. It is shown that because of the interdimensional degeneracy of eigenvalues, we can also reproduce eigenvalues of a upper/lower dimensional system from the well-known eigenvalues of a lower/upper dimensional system by means of the transformation (n, l, D) -¿ (n, l +/- 1, D +/- 2). This solution reduces to the Hulthen potential case.

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Abstract: A new non-central potential, consisting of a pseudoharmonic potential plus another recently proposed ring-shaped potential, is solved. It has the form V(r, theta) = 1/8 Kr-e(2) (r/r(e) - r(e)/r)(2) + beta cos(2)theta/r(2)sin(2)theta. The energy eigenvalues and eigenfunctions of the bound-states for the Schrodinger equation in D-dimensions for this potential are obtained analytically by using the Nikiforov-Uvarov method. The radial and angular parts of the wave functions are obtained in terms of orthogonal Laguerre and Jacobi polynomials. We also find that the energy of the particle and the wave functions reduce to the energy and the wave functions of the bound-states in three dimensions.

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Abstract: We present analytically the exact energy bound-states solutions of the Schrodinger equation in D dimensions for a recently proposed modified Kratzer plus ring-shaped potential by means of the Nikiforov-Uvarov method. We obtain an explicit solution of the wave functions in terms of hyper-geometric functions (Laguerre polynomials). The results obtained in this work are more general and true for any dimension which can be reduced to the well-known three-dimensional forms given by other works.

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Abstract: In the framework of static and QCD-motivated model potentials for heavy quarkonium, we present a further comprehensive calculation of the mass spectrum of bc system and its ground state spin-dependent splittings in the context of the shifted l-expansion technique. We also predict the leptonic constant fB(c), of the lightest pseudoscalar B,, and fB(c)*. of the vector B-c* states taking into account the one-loop and two-loop QCD corrections. Furthermore, we use the scaling relation

to predict the leptonic constant of the nS-states of the bc system. Our predicted results are generally in high agreement with some earlier numerical methods. The parameters of each potential are adjusted to obtain best agreement with the experimental spin-averaged data (SAD).

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Abstract: We present the exact solution of the Klein Gordon equation in D-dimensions in the presence of the equal scalar and vector pseudoharmonic potential plus the ring-shaped potential using the Nikiforov-Uvarov method. We obtain the exact bound state energy levels and the corresponding eigen functions for a spin-zero particles. We also find that the solution for this ring-shaped pseudoharmonic potential can be reduced to the three-dimensional (3D) pseudoharmonic solution once the coupling constant of the angular part of the potential becomes zero.

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Abstract: For arbitrary values of n and l quantum numbers, we present a simple exact analytical solution of the D-dimensional (D a parts per thousand yen 2) hyperradial Schrodinger equation with the Kratzer and the modified Kratzer potentials within the framework of the exact quantization rule (EQR) method. The exact bound state energy eigenvalues (E (nl)) are easily calculated from this EQR method. The corresponding normalized hyperradial wave functions (psi (nl) (r)) are also calculated. The exact energy eigenvalues for these Kratzer-type potentials are calculated numerically for a few typical LiH, CH, HCl, CO, NO, O-2, N-2 and I-2 diatomic molecules for various values of n and l quantum numbers. Numerical tests using the energy calculations for the inter dimensional degeneracy (D = 2 - 4) for I-2, LiH, HCl, O-2, NO

and CO are also given. Our results obtained by EQR are in exact agreement with those obtained by other methods.

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Abstract: The 1/d expansion method for polymer chains is examined by comparing these expansions for several thermodynamic and structural quantities with the results of standard series analysis of exact enumeration data. The comparisons cover a wide range of spatial dimensions d, including non-integer ones, and are performed for particular values of interaction energy. Good agreement is generally found for d¿4, whereas discrepancies become conspicuous as d decreases to d= 2. Reasonable values are obtained for the exponents nu

and gamma in d = 2 - 4 by applying the coherent-anomaly method of Suzuki to our 1/d expansions through fifth order in d-1.

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Abstract: We study the renormalization of the O(N) model using the auxiliary field formalism (Hubbard-Stratonovich transformation) in the 1/N expansion at finite temperature. We provide the general strategy of renormalization for arbitrary order and make a calculation up to next-to-leading order. We show that renormalization is possible for any values of the condensates, prove the temperature independence of the counterterms, and determine the cutoff dependence of the first nontrivial counterterm parts.

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Abstract: Using dimensional scaling, we were able to obtain a systematic expansion for Regge trajectories in 1/kappa, where kappa = (D-1)/2 and D is the number of spatial dimensions. Bound states for the power-law potential were obtained from Regge trajectories by requiring the angular momentum quantum number to take on positive integer values. For scattering states, we calculated the positions of Regge poles for the Lennard-Jones(6,4) and (12,6)potentials. The results to first order in 1/kappa were in good agreement with both semiclassical and quantum calculations. The same expansion was used to obtain the positions of Regge poles for complex optical potentials. The results for the Lennard-Jones (I 2,6) potential perturbed by an imaginary term were in excellent agreement with the semiclassical calculations.

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Abstract: We derive a simple, analytic expression for the energy splitting DELTA-E between

the lowest pair of H2+ states (1s-sigma(g) and 2p-sigma(u)) that results from electron exchange between two protons. The calculation employs the semiclassical instanton method, with two unorthodox features which markedly simplify the treatment: (1) The double-minimum potential and corresponding wavefunctions that govern the electronic tunneling are evaluated in the large-dimension limit. (2) The time variable is rescaled to cure divergent behavior of fluctuations about the instanton path that otherwise appears because of the potential develops sharp cusps as the internuclear distance increases. By virtue of exact interdimensional degeneracies, the large-D limit yields valid results for a 3D molecule. Indeed, a simple dimensional scaling law gives DELTA-E for all pairs of g, u states that stem from separated atom states with m = l = n - 1, for n = 1 –¿ infinity. For a wide range of internuclear distances, our analytic expression for DELTA-E, which pertains to the leading order in HBAR, gives for such states good agreement with comparable semiclassical methods as well as with exact numerical calculations. It is remarkable that use of the effective potential for the large-dimension limit, which is exactly calculable from classical electrostatics, yields quantitative results for electronic tunneling, an intrinsically quantal phenomenon.

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Abstract: Analytic expressions for the D –¿ infinity limit previously evaluated for one- and two-electron diatomic molecules are compared with the familiar D = 3 potentials as functions of the internuclear distance R for H-2(+), H-2, HHe+, and He-2(2+). We find that the D = 3 potential can be obtained from the large-D limit simply by E(3)(R) E(infinity)(R)/G(R), to good approximation, where the function G(R) has the same form for the four molecules. Furthermore, 1 - G(R) has a corresponding-states property; when reduced by two parameters, it becomes the same function for

all four molecules, nearly proportional to x(2)e(-2x), with x the scaled internuclear distance. This suggests that the pseudoclassical D –¿ infinity limit may provide a useful reference for analysis of intermolecular potentials.

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Abstract: Complex energy eigenvalues which specify the location and width of quasibound or resonant states are computed to good approximation by a simple dimensional scaling method. As applied to bound states, the method involves minimizing an effective potential function in appropriately scaled coordinates to obtain exact energies in the D –¿ infinity limit, then computing approximate results for D = 3 by a perturbation expansion in I/D about this limit. For resonant states, the same procedure is used, with the radial coordinate now allowed to be complex. Five examples are treated: the repulsive exponential potential (e(-r)); a squelched harmonic oscillator (r2e(-r)); the inverted Kratzer potential (r-1 repulsion plus r-2 attraction); the Lennard-Jones potential (r-12 repulsion, r-6 attraction); and quasibound states for the rotational spectrum of the hydrogen molecule (X 1SIGMA(g)+, v = 0, J = 0 to 50). Comparisons with numerical integrations and other methods show that the much simpler dimensional scaling method, carried to second-order (terms in 1/D2), yields good results over an extremely wide range of the ratio of level widths to spacings. Other methods have not yet evaluated the very broad H-2 rotational resonances reported here (J¿ 39), which lie far above the centrifugal barrier.

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Abstract: Analytic expressions for the large-dimension limit, when renormalized by introducing:a suitable effective nuclear charge zeta yield

accurate D=3 nonrelativistic energies for ground states of many-electron atoms. Using Hartree-Fock data to estimate zeta, which typically differs from the actual charge Z by similar to 1% or less, we find this dimensional renormalization method (denoted DR-O) gives results substantially better than the HF input. Comparison of the 1/Z expansion for the large-D limit with that for D=3 atoms provides expressions for the leading error terms in the renormalized total energy and correlation energy. When configuration;mixing occurs in the Z–¿infinity limit (as for Be and many other atoms), we find the renormalization procedure is markedly improved by including the zeroth-order mixing (denoted DR-1); this-contributes-a term linear in Z. Including the Z-independent term (DR-2) also improves the accuracy when zeroth-order mixing is absent (e.g., ground-state atoms with N=2, 3, and 7-11) but not otherwise. Correlation energies for atoms and cations with N=2-18 electrons and Z=2-28 are obtained with a mean error of 26% using just the large-D limit or HF input (DR-O); the mean error improves to only 5% when the leading 1/Z term is included (either DR-1 or DR-2). Results much better than the HF approximation are likewise obtained for the ionization potentials and electron affinities of neutral atoms.

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exchange, and correlation potentials and energies with results from two approximations. These use Becke's exchange functional and either the Lee-Yang-Parr or the Perdew correlation functional. Both approximations yield rather good results for the density and the exchange and correlation energies, but both deviate markedly from the exact exchange and correlation potentials. We also compare properties of the Hooke's Law model with those of two-electron atoms, including the large dimension limit. A renormalization procedure applied to this very simple limit yields correlation energies as good as those obtained from the approximate functionals, for both the model and actual atoms.

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Abstract: Dimensional scaling generates an effective potential for the electronic structure of atoms and molecules, but this potential may acquire multiple minima for certain ranges of nuclear charges or geometries that produce symmetry breaking. Tunneling among such minima is akin to resonance among valence bond structures. Here we treat the D-dimensional H-2+ molecule ion as a prototype test case. In spheroidal coordinates it offers a separable double-minimum potential and tunneling occurs in only one coordinate; in cylindrical coordinates the potential is nonseparable and tunneling occurs in two coordinates. We determine for both cases the ground state energy splitting DELTA-E(D) as a function of the internuclear distance R. By virtue of exact interdimensional degeneracies, this yields the exchange energy for all pairs of g, u states of the D = 3 molecule that stem from separated atom states with m = l = n - 1, for n = 1 –¿ infinity. We evaluate DELTA-E(D) by two semiclassical techniques, the asymptotic and instanton methods, and obtain good agreement with exact numerical calculations over a wide range of R. We

find that for cylindrical coordinates the instanton path for the tunneling trajectory differs substantially from either a straightline or adiabatic path, but is nearly parabolic. Path integral techniques provide relatively simple means to determine the exact instanton path and contributions from fluctuations around it. Generalizing this approach to treat multielectron tunneling in several degrees of freedom will be feasible if the fluctuation calculations can be made tractable.

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Abstract: Quantum phase transitions at absolute zero temperature can take place as some parameter in the Hamiltonian of the system is varied. For such transitions, crossing the phase boundary means that the quantum ground state changes in some fundamental way. For the Hamiltonian of N-electron atoms, this parameter is taken to be the nuclear charge. As the nuclear charge reaches a critical point, the quantum ground state changes its characters from being bound to being degenerate or absorbed by a continuum. We describe here a method to calculate the critical nuclear charge for which an atom can bind an extra electron to form a stable negative ion. The estimate of the critical nuclear charge will be used to explain and predict the stability of atomic negative ions. The method can be generalized to predict the stability of molecular negative ions. A detailed calculation for the critical parameters for two center molecular ions is also included. (C) 1999 Elsevier Science B.V.

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Schrodinger equation. In this approach, the finite size corresponds to the number of elements in a complete basis set used to expand the exact eigenfunction of a given Hamiltonian. The critical parameters such as the critical nuclear charges will be used to explain and predict the stability of atomic and molecular negative ions. For N-electron atoms with 2 less than or equal to N less than or equal to 86, results show that, at most, only one electron can be added to a free atom in the gas phase. However, doubly charged atomic negative ions might exist in a strong magnetic field.

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Abstract: By combining Hartree-Fock results for nonrelativistic ground-state energies of N-electron

atoms with analytic expressions for the large-dimension limit, we have obtained a simple renormalization procedure. For neutral atoms, this yields energies typically threefold more accurate than the Hartree-Fock approximation. Here, we examine the dependence on Z and N of the renormalized energies E(N, Z) for atoms and cations over the range Z, N = 2 –¿ 290. We find that this gives for large Z = N an expansion of the same form as the Thomas-Fermi statistical model, E –¿ Z(7/3)(Co + C(1)Z(-1/3) + C(2)Z(-2/3) + C(3)Z(-3/3) + ...), with similar values of the coefficients for the three leading terms. Use of the renormalized large-D limit enables us to derive three further terms. This provides an analogous expansion for the correlation energy of the for Delta E –¿ Z(4/3)(Delta C-3 + Delta C(5)Z(-2/3) + Delta C(6)Z(-3/3) + ...); comparison with accurate values of Delta E available for the range Z less than or equal to 36 indicates the mean error is only about 10%. Oscillatory terms in E and Delta E are also evaluated. (C) 1994 John Wiley gr Sons, Inc.

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Abstract: The semiclassical approximation and the technique of 1/n expansion are used to calculate the eigenenergies and the wave functions for the radial Schrodinger equation. It is shown that the expressions that are asymptotically exact in the limit n = n(r) + l + 1 –¿ infinity and which describe the above eigenenergies and the asymptotic coefficients at the origin and at infinity ensure a satisfactory precision even for states characterized by modest values of the quantum numbers II, and I, including the ground state. (C) 2001 MAIK "Nauka/Interperiodica".

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Abstract: A method is developed for calculating wave functions in the subbarrier region on the basis 1/n expansion. This method is used to calculate the asymptotic coefficient c(lp) at the origin. To illustrate the accuracy of the method the results are compared with results of numerical calculations for power-law potentials, short-range Yukawa potentials, and exponential potentials. Calculations ar performed for node-free states and for states with radial quantum number p = 1. The calculated correction of order 1/n is shown to considerably increase the accuracy of 1/n expansion, especially for p = 1 states (at n similar to 1, a zero-order approximation for such states is only an estimate).

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supersymmetry, for which the two supercharge operators are obtained in terms of (a D-dimensional generalization of) the Johnson-Lippmann operator, an extension of the Runge-Lenz-Pauli vector that relativistically incorporates spin degrees of freedom. So the extra symmetry (S(2)) in the quantum Kepler problem, which determines the degeneracy of the levels, is so robust as to accommodate the relativistic case in arbitrary dimensions. (c) 2006 American Institute of Physics.

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physical level of rigor: (1) a large number of PT coefficients are known whose asymptotics has already been established, and (2) there is no intermediate asymptotics. Higher orders of PT, their asymptotic behavior, and power corrections are calculated in quantum mechanical problems that involve divergent PT series (including series for a funnel potential, the phi((0))(4) model, and the Stark effect in a strong field). The scalar field theory phi((4))(4) is considered in the (MS) over bar and MOM regularization schemes. It is shown that one cannot make any definite conclusion about the asymptotics of the Gell-Mann-Low function as g –¿ infinity on the basis of information available for the above theory. (C) 2002 MAIK "Nauka/Interperiodica".

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Abstract: Confined quantum systems involving N identical interacting particles are to be found in many areas of physics, including condensed-matter, atomic, and chemical physics. A beyond-mean-field perturbation method that is applicable, in principle, to weakly, intermediate, and strongly interacting systems has been set forth by the authors in a previous series of papers. Dimensional perturbation theory was used, and in conjunction with group theory, an analytic beyond-mean-field correlated wave function at lowest order for a system under spherical confinement with a general two-body interaction was derived. In the present

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physical subspace, a feature which significantly enhances the ability of a localized structure to represent the true solution and to model it quantitatively. Specific calculations reported are total and correlation energies for atoms with Z less-than-or-equal-to 92 (computed by post-scaling and pre-scaling), correlation corrections to ionization potentials and electron affinities for Z less-than-or-equal-to 54 (post-scaling), and the asymptotic behavior of correlation energies for Z –¿ infinity (post-scaling).

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a third-quantized form. The third-quantized creation and annihilation operators depend on the field phi(x). If the coefficient of the phi(4) term is of order 1/N (this is the usual condition of applicability of the 1/N expansion), one can rescale the third-quantized operators in such a way that their commutator will be small, while the Heisenberg equations will not contain large or small parameters. This means that the classical equation of motion is an equation on the functional Phi[phi(.)]. This equation, being a nonlinear analogue of the functional Schrodinger equation for the one-field theory, is investigated. The exact solutions are constructed and the renormalization problem is analyzed. We also perform a quantization procedure about found classical solutions. The corresponding semiclassical theory is a theory of a variable number of fields. The developed third-quantized semiclassical approach is applied to the problem of finding the large-N spectrum. The results are compared with formulas obtained by known methods. We show that not only the known but also new energy levels can be found. [S0556-7821(99)00613-8].

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two-body correlations at all orders and yields analytical results through first order by taking advantage of the high degree of symmetry of the condensate at the zeroth-order limit. Because N remains a parameter in our analytical results, the challenge of calculating energies and excitation frequencies does not rise with the number of condensate atoms. In this proof-of-concept paper the atoms are confined in a spherical trap and are treated as hard spheres. Our many-body calculations compare well to Gross-Pitaevskii results in the weakly interacting regime and depart from the mean-field approximation as the density approaches the strongly interacting regime. The excitation frequencies provide a particularly sensitive test of beyond-mean-field corrections. For example, for N=2000 atoms and an experimentally realized large scattering length of a=0.433a(ho) (a(ho)=root(h) over bar /momega(ho)) we predict a 75% shift from the mean-field breathing mode frequency.

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interdimensional degeneracy of the confined D-dimensional (D ¿ 1) harmonic oscillator energy levels are derived, assuming that the isotropic confinement is defined by an infinite potential well and a finite radius R-c. Very accurate energy eigenvalues are obtained numerically by finding the roots of the confluent hypergeometric functions that confirm the degeneracy conditions. (c) 2006 Wiley Periodicals, Inc.

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knownthree-dimensional solutions of the two problems. Using the known ground-state wave functions for these systems, we construct supersymmetric partner potentials that allow us to apply the supersymmetric large-Nexpansion to obtain accurate approximate energy eigenvalues.

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Abstract: Perturbation theory (to the first order) for quasistationary states is developed in the semiclassical approximation. Formulas for the energy shift delta E, of a resonance, as well as for the variation of its width delta Gamma due to a perturbative potential delta U, are obtained. These formulas are valid both for subbarrier and above barrier resonances. The results are illustrated using a number of potentials for which semiclassical expressions can be compared with exact solutions of the Schrodinger equation. The proposed scheme is generalized to a multidimensional case with completely separating variables.

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Abstract: A generalization of the Gamow formula for the width GAMMA of a quasistationary level (with energy E = E(r) - i-GAMMA/2) is given for the case of multidimensional systems with separable variables. The condition for applicability of this approximation is obtained, and some examples are considered.

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Abstract: The shifted-1/N-expansion method has been used to study the energy levels for a 2D donor impurity in the presence of an arbitrary magnetic field. Exact analytical expressions for the energy levels at zero- and high-field limits are obtained. The calculations are carried out to the third-order correction of the shifted-1/N-expansion energy series. The results appear to be in excellent agreement with those of Martin et al.

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the Q-vector Coulomb potential in both KG and Dirac equations. Highly accurate and fast converging results are obtained for the scalar linear and the equally mixed scalar and 4-vector power-law potentials.

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Abstract: The d-dimensional generalization of the point canonical transformation for a quantum particle endowed with a position-dependent mass in the Schrodinger equation is described. Illustrative examples including the harmonic oscillator, Coulomb, spiked harmonic, Kratzer, Morse oscillator, Poschl-Teller and Hulthen potentials are used as reference potentials to obtain exact energy eigenvalues and eigenfunctions for target potentials at different position-dependent mass settings.

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Abstract: The energy levels of neutral atoms supported by potential V(r) = -Z exp(-alphar)/r (Yukawa potential) are studied, using both dimensional and dimensionless quantities, via a new analytical methodical proposal (devised to solve for nonexactly solvable Schrodinger equation). Using dimensionless quantities, by scaling the radial Hamiltonian through y Zr and alpha ' = alpha /Z, we report that the scaled screening parameter alpha ' is restricted to have values ranging from zero to less than 0.4. On the other hand, working with the scaled Hamiltonian enhances the accuracy and extremely speeds up the convergence of the energy eigenvalues. The energy levels of several new eligible scaled screening parameter alpha ' values are also reported.

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Abstract: A different approach to the shifted 1/N expansion technique is developed to deal with the Klein-Gordon particle trapped in a spherically symmetric potential. Properly modifying the definition of the perturbative expansion of the energy eigenvalue, and without making any approximation in the determination of the parameters involved, we obtain sufficiently good results compared with the exact ones for the Coulomb problem. The calculations are carried out to the second-order correction of the energy series.

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Abstract: The shifted 1/N expansion method has been extended to solve the Klein-Gordon equation

with both scalar and vector potentials. The calculations are carried out to the third-order correction in the energy series. The analytical results are applied to a linear scalar potential to obtain the relativistic energy eigenvalues. Our numerical results are compared with those obtained by Gunion and Li [Phys. Rev. D 12, 3583 (1975)].

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Abstract: A different approach to the shifted 1/N expansion method is developed to deal with the Dirac particle trapped in a spherically symmetric potential. The main aspects of our approach are to expand the energy term in a perturbative form and to determine the parameters involved without any approximation. While the formalism is developed for spin-1/2 particles in any spherically symmetric potential, it is applied to the Coulomb case for testing. The calculations are carried out to the third-order correction of the energy eigenvalue series.

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Abstract: A mapping between the quantum few-body problem and its classical mechanics pseudo-

system analog is used to study the critical parameters for the helium isoelectronic sequence. The critical point is the critical value of the nuclear charge Z(c), for which the energy of a bound state becomes degenerate with a threshold. A finite-size scaling ansatz in the form of a phenomenological renormalization equation is used to obtain very accurate results for the critical point of the ground-state energy, lambda(c) = 1/Z(c) = 1 .0976+/-0.0004, as well as for the excited 2p(2) P-3 state, lambda(c) = 1.0058+/-0.0017. The results for the critical exponents alpha and nu are also included. (C) 1998 American Institute of Physics.

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Abstract: In modern fundamental theories there is consideration of higher dimensions, often in the context of what can be written as a Schrodinger equation. Thus, the energetics of bound states in different dimensions is of interest. By considering the quantum square well in continuous D dimensions, it is shown that there is always a bound state for 0 ¡ D less than or equal to 2. This binding is complete for D –¿ 0 and exponentially small for D –¿ 2-. For D ¿ 2, a finite-sized well is always needed for there to be a bound state. This size grows like D-2 as D gets large. By adding the proper angular momentum tail, a volcano, zero-energy, bound state can be obtained. (C) 2002 Elsevier Science B.V. All rights reserved.

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appears as a continuous variable. We discuss implications for quantum Monte Carlo and dimensional scaling methods. (c) 2005 American Institute of Physics.

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> **Abstract:** We study a PT-symmetric quantum-mechanical model with an O(N)-symmetric potential of the form m(2)(x) over right arrow (2)/2 - g((x) over right arrow (2))(2)/N using its equivalent Hermitian form. Although the corresponding classical model has finite-energy trajectories that escape to infinity, the spectrum of the quantum theory is proven to consist only of bound states for all N. We show that the model has two distinct phases in the large-N limit, with different scaling behaviors as N goes to infinity. The two phases are separated by a first-order phase transition at a critical value of the dimensionless parameter m(2)/g(2/3), given by 3 x 2(1/3).

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> **Abstract:** The hypervirial and Hellmann-Feynman theorems are used in the methods of 1/N expansion to construct Rayleigh-Schrodinger perturbation expansion for bound-state energy eigenvalues of spherical symmetric potentials. An iteration procedure of calculating correction terms of arbitrarily high orders is obtained for any kind of 1/N expansion. The recurrence formulas for three variants of the 1/N expansion are considered in this work. namely? the 1/n expansion and the shifted and unshifted 1/N expansions which are applied to the Gaussian and Patil potentials. As a result. their credibility could be reliably judged when account is taken of high-order terms of the eigenenergies. It is also found that there is a distinct advantage in using the shifted 1/N expansion over the two other versions. However, the shifted 1/N expansion diverges for s states and in certain cases is not

applicable as far as complicated potentials are concerned. In an effort to solve these problems we have incorporated the principle of minimal sensitivity in the shifted 1/N expansion as a first step toward extending the scope of applicability of that technique, and then we have tested the obtained approach to some unfavorable cases of the Patil and Hellmann potentials. The agreement between our numerical calculations and reference data is quite satisfactory.

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Abstract: A new empirical potential energy function for diatomic molecules, which has the simple form of V(r) = (ar(b) + m)/(1 - e(nr)) is introduced, where a, b, m and n are variational parameters of the function. To obtain the parameters of this function, it was fitted to the points, which were obtained previously from the RKR calculations for a wide range of diatomic molecules in both their ground and excited states. The reliability of the proposed function was checked by calculation of some spectroscopic constants such as bond length (R-e), dissociation energy (D-e), force constant (K-e), rotational constant (B-e), vibrational frequency (omega(e)), anharmonicity constant (omega(c)x(c)) and vibration-rotation coupling constant (alpha(e)) for these molecules. For omega(e)x(e) and alpha(e) the results were compared with those of the Morse, Rosen-Morse, Rydberg, Poschl-Teller, Linnett. Frost-Muslin, Varshni, Lippincott and Rafi functions. Our results are consistent with or have less error than those of the above-mentioned functions.

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Abstract: In this work we present a theoretical study of the d-dimensional Coulomb problem in quantum phase space. A coordinate transformation in hyperspherical space is used that maps the d-dimensional Coulomb problem into

the D-dimensional harmonic oscillator and the Wigner distribution function for the d-dimensional Coulomb problem is then obtained. This exactly soluble model can shed some light on finite-size features of Wigner's distribution, which will be a vital experience for various dynamic problems. (C) 1999 American Institute of Physics. [S0022-2488(99)03502-1].

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Abstract: In this paper a set of generalized coherent states for the d-dimensional Coulomb problem in coordinate representation are constructed. A coordinate transformation in hyperspherical space is used that maps the d-dimensional Coulomb problem into the D-dimensional harmonic oscillator and the generalized coherent states for the d-dimensional Coulomb problem are then obtained. This exactly soluble model can provide an adequate means for a quantum coherency description of the Coulomb problem in arbitrary dimensions, specifically in the special case of the hydrogen atom, in many theoretical and applied related fields such as in coherent scattering. [S1050-2947(99)05008-8].

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Abstract: A perturbation approach to compute the bound states of the Schrodinger equation HPsi=EPsi with H-0+lambdaV and Psiparallel to(x=+/-infinity)=0 is studied. The approach involves solving the corresponding Dirichlet problem H(R)Psi(R)=E(R)Psi(R) on a finite interval [-R,R] by the Rayleigh-Schrodinger perturbation theory (RSPT). The method is based on the fact that E-R,Psi(R) converge to E,Psi as R–¿infinity. The model problems to study the summability properties of the RSPT series E-R=Sigma(k=0)(infinity)E(R)((k))lambda(k) are the anharmonic oscillators H=p(2)+x(2)+lambdax(2M), with M=2,3,4 for which the RSPT produces strongly divergent

series E=Sigma(k=0)(infinity)E((k))lambda(k). The summation of the latter series with large lambda for the octic case is considered as an extremely challenging summation problem, in part, since it was rigorously proven that the Pade approximants cannot converge and the two-point Pade approximants, which combine information of the renormalized weak coupling and strong coupling expansions, give relatively good results. The calculations of this work show that the ordinary Pade approximants from the sole unnormalized E-R series for the octic oscillator give accurate results with small or large lambda. The coefficients E-R((k)) are calculated with the eigenvalue series of an operator H-Rn, whose resolvent converges to that of H-R as n–¿infinity. The Pade approximants of the RSPT eigenfunction series Psi(R)=Sigma(k=0)(infinity)psi(R)((k))lambda(k) also provide accurate results for the octic oscillator.

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Abstract: The periodic Anderson model with strong repulsion is studied. The leading-order effects with respect to 1/d, d being the dimensionality of the system, are Kondo-effect-type local spin fluctuations and mean-field-approximation-type magnetic orders. The competition between the Kondo effect and the Ruderman-Kittel-Kasuya-Yosida exchange interaction determines whether the ground state is magnetic or paramagnetic. A single-site approximation (SSA) is rigorous for paramagnetic states in d –¿ + infinity dimensions. The SSA reduces the problem to solving the Anderson model. A "local" Kondo temperature is defined to show an energy scale of the local spin fluctuations. On the other hand, an SSA including magnetic mean fields is rigorous for magnetic states in d –¿ + infinity dimensions. All the other effects are of higher order: Critical and intersite spin fluctuations are O(1/square-root d); and paramagnetic orders are at most O(1/d). The 1/d expansion is one of the most useful methods for examining lower-temperature phases in real dimensions.

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> **Abstract:** We derived analytical expressions for the energy of classical Circular Rydberg States (CRS) in collinear electric (F) and magnetic (B) fields of arbitrary strengths. Previously published explicit expressions for the energy E were given only for the region of a weak electric field F and only in the limits of B –¿ 0 and B –¿ infinity. We offered formulas for the dependence of the classical ionization threshold F-c(B) and of the energy at this threshold E-c(B) valid for the magnetic field B of an arbitrary strength. We also analyzed the stability of the motion by going beyond the CRS. In addition, for two important particular cases previously studied in the literature – classical CRS in a magnetic field only and classical CRS in an electric field only – we presented some new results as well.

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> **Abstract:** Some aspects of the N dimensional Kratzer-Fues potential are discussed, which is an extension of the combined Coulomb-like potential with inverse quadratic potential in N dimensions. The analytical solutions obtained (eigenfunctions and eigenvalues) are dimensionally dependent, so also, the solutions depend on the value of the coefficient of the inverse quadratic term. The expectation values for ¡ r(-2) ¿, ¡ r(-1) ¿ and the virial theorem for this potential are obtained and the values are also dimensions and parameter dependent.

[461] K. J. Oyewumi, F. O. Akinpelu, and A. D. Agboola. Exactly complete solutions of the pseudoharmonic potential in N-dimensions. *Int. J. Theor. Phys.*, 47(4):1039–1057, 2008.

Abstract: We present analytically the exact solutions of the Schrodinger equation in the N-dimensional spaces for the pseudoharmonic oscillator potential by means of the ansatz method. The energy eigenvalues of the bound states are easily calculated from this eigenfunction ansatz. The normalized wavefunctions are also obtained. A realization of the ladder operators for the wavefunctions is studied and we deduced that these operators satisfy the commutation relations of the generators of the dynamical group SU(1,1). Some expectation values for ¡ r(-2)¿, ¡ r(2)¿, ¡ T ¿, ¡ V ¿, ¡ H ¿, ¡ p(2)¿ and the virial theorem for the pseudoharmonic oscillator potential in an arbitrary number of dimensions are obtained by means of the Hellmann-Feynman theorems. Each solution obtained is dimensions and parameters dependent.

[462] K. J. Oyewumi and E. A. Bangudu. Isotropic harmonic oscillator plus inverse quadratic potential in N-dimensional spaces. *Arab. J. Sci. Eng.*, 28(2A):173–182, 2003.

Abstract: Some aspects of the N-dimensional isotropic harmonic plus inverse quadratic potential are discussed. The hyperradial equation for isotropic harmonic oscillator Plus inverse quadratic potential is solved by transformation into the confluent hypergeometric equation to obtain the normalized hyperradial Solution. Together with the hyperangular solutions (hyperspherical harmonics), these form the complete energy eigenfunctions of the N-dimensional isotropic harmonic oscillator plus inverse quadratic potential and the energy eigenvalues are also obtained. These are dimensionally dependent. The dependence of the radial solution on the dimensions or the potential strength and the degeneracy of the energy levels are discussed.

[463] S. Panchanan, R. Roychoudhury, and Y. P. Varshni. Modified shifted 1/N expansion for relativistic Yukawa and Hulthen potentials. *Can. J. Phys.*, 74(3-4):136–140, 1996.

Abstract: The shifted 1/N expansion technique is applied to the radial Dirac equation. it is shown that the fixing condition E(1) = 0 produces an enhanced accuracy for Yukawa and Hulthen potentials. The modified shifted 1/N expansion results

are compared with the exact numerical results and results obtained by other approximate methods.

[464] M. M. Panja, M. Bag, R. Dutt, and Y. P. Varshni. LARGE-N EXPANSION METHOD FOR A SPIN-1/2 PARTICLE IN THE PRESENCE OF VECTOR AND SCALAR POTENTIALS. *Phys. Rev. A*, 45(3):1523–1530, 1992.

Abstract: The shifted large-N technique (SLNT) has been applied to study the relativistic motion of a particle in the presence of vector and scalar interactions with special emphasis on the construction of both large- and small-component Dirac radial wave functions. Numerical results for the binding energy for a particle in the presence of the Coulomb plus linear confining potential compare very well with those obtained by the elaborate analytic approximation method using the Pade-approximation technique. We illustrate that one recovers not only the exact analytic results for binding energies for vector and scalar Coulomb potentials, but also exact wave functions from the leading-order SLNT calculation. This motivates future applications of the same method to more realistic atomic systems governed by screened Coulomb potentials where the knowledge of the large and small components of the radial wave function is essential.

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[477] E. Papp. QUASI-CLASSICAL 1/N ENERGY FORMULAS FOR THE SUPERPOSITION BETWEEN THE ATTRACTIVE POTENTIAL -LAMBDA-0/X2 AND THE REPULSIVE (N-GREATER-THAN-2)-POTENTIAL -GAMMA(N)/XN. *Europhys. Lett.*, 9(4):309–313, 1989.

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> **Abstract:** Quickly tractable energy-formulas for relativistic two-body systems are established to first 1/N-order in terms of general classical Hamiltonians (m1 greater-than-or-equal-to m2). Arbitrary spherically-symmetrical vector and/or scalar

potentials are assumed. Such Hamiltonians, which have to be converted into related non-relativistic ones, play the role of a dominant zeroth-order approximation (an exact description) in the case of two fermions (two bosons). Proofs have also been given that the 1/N-method is quite suitable for the systematic description of order-dependent contributions to the energy of Coulomb-Bethe-Salpeter systems, now to alpha-6-order.

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Abstract: Energy formulae for two-particle Klein-Gordon systems subjected to Coulomb and Yukawa potentials are derived to first 1/N-order. The energies obtained in this manner incorporate the influence of the mass-asymmetry via the parameter-delta = (m1(2) - M2(2))2. Certain Coulomb-inspired extrapolations to Dirac-particle systems, having the meaning of dominant contributions, are taken into account. Numerical results are presented.

[480] E. Papp. CLOSED 1/N MASS FORMULAS FOR RELATIVISTIC 2-BODY Q1QBAR2 SYSTEMS. *Phys. Lett. B*, 259(1-2):19–23, 1991.

Abstract: Proofs are given that the mass spectra of bsBAR mesons predicted by Crater and Van Alstine can be reproduced, within relative errors smaller than a few percents, by applying simplified 1/N equations, now just in terms of a related phenomenological vectorial Coulomb plus linear potential. Such equations are also useful for a quickly tractable energy description of other relativistic two-body systems with arbitrary masses, for which the constituent Coulomb potential plays a significant role.

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Abstract: A general q-deformation formula for the expansion parameter of the 1/N method

is derived. One proceeds by interpolating between the suitable q-deformed energy of the three-dimensional Coulomb problem and the symmetrized q-deformed energy of the harmonic oscillator in N space-dimensions. Useful q-deformed 1/N energy formulae for arbitrary spherically symmetrical O(N)-potentials are presented.

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Abstract: The 1/N approach to the Harper equation proposed previously is generalized towards performing the pertinent band-energy description. Leading forms and corrections proceeding to first order in the magnetic field are written down. The energy reflection symmetry has also been discussed.

[484] E. Papp and C. Micu. 1/N-ENERGY-SOLUTIONS TO RELATIVISTIC 2-BOSON SYSTEMS WITH AN EXPONENTIAL POTENTIAL. *Z. Naturfors. Sect. A-J. Phys. Sci.*, 46(8):729–732, 1991.

Abstract: Closed 1/N-energy-formulae for relativistic two-boson (m1 not-equal m2) systems with a vector exponential potential have been written down to first 1/N-order. Numerical estimates and structural properties are presented.

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Abstract: Proofs are given that SOq(N)-motivated q deformations of the expansion parameter of the 1/N method can be incorporated into symmetry transformations preserving the radial form of the Schrodinger equation. This opens the way to establish the q deformed energy and the corresponding 1/N equivalent potential. So far the number of space dimensions is subject of covariant transformations. An alternative derivation of

the 1/N equivalent potential is also done by keeping this number invariant. In this latter case, the stability attributes of q-deformed power-law potentials are discussed in terms of the q¿1 choice.

[486] G. Parisi and F. Slanina. On the 1/D expansion for directed polymers. *Eur. Phys. J. B*, 8(4):603–611, 1999.

Abstract: We present a variational approach for directed polymers in D transversal dimensions which is used to compute the correction to the mean field theory predictions with broken replica symmetry. The trial function is taken to be a symmetrized version of the mean-field solution, which is known to be exact for D = infinity. We compute the free energy corresponding to that function and show that the finite-D corrections behave like D-4/3. It means that the expansion in powers of 1/D should be used with great care here. We hope that the techniques developed in this note will be useful also in the study of spin glasses.

[487] S. H. Patil. Hydrogen molecular ion and molecule in two dimensions. *J. Chem. Phys.*, 118(5):2197–2205, 2003.

Abstract: We discuss some general properties of the wave functions of H-2(+) in two dimensions, when the electron is close to the nucleus, and when it is far away from the nucleus. Based on these properties, some simple, parameter-free wave functions are developed for some of the states. They lead to quite accurate energies and give an insight into the structure of H-2(+) in two dimensions. These wave functions and appropriate correlation functions, are then used to develop reliable wave functions for H-2 in two dimensions. They provide a clear and interesting comparison with the corresponding results in three dimensions. (C) 2003 American Institute of Physics.

[488] S. H. Patil. The helium atom and isoelectronic ions in two dimensions. *Eur. J. Phys.*, 29(3):517–525, 2008.

Abstract: The energy levels of the helium atom and isoelectronic ions in two dimensions are considered. The difficulties encountered in the analytical evaluation of the perturbative and variational

expressions for the ground state, promote an interesting factorization of the inter-electronic interaction, leading to simple expressions for the energy. This expression provides an insight into the general structure and screening effect in the ground state. Some of the considerations are extended to excited states. The dimensionality properties of the energy spectrum of a helium atom, the screening effect and their implications are of significant pedagogical interest in the study of perurbative and variational approaches to analyse the properties of helium atoms, in quantum mechanics and atomic physics, for general graduate students and specialists in atomic physics.

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Abstract: Some structural properties of energy eigenfunctions of two electrons in a simple harmonic potential are analyzed. Simple expressions are obtained for the energy spectrum based on a model potential, and on a model wave function. These expressions give accurate values for the energy eigenvalues and provide a physical insight into their structure.

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Abstract: In the large-N-c limit of QCD, the expansion of the vacuum polarization at low energies determines the whole function at any arbitrarily large (but finite) energy. This result is an immediate consequence of the theory of Pade approximants to Stieltjes functions.

[491] S. Peris, M. Perrottet, and E. de Rafael. Matching long and short distances in large-N-c QCD. *J. High Energy Phys.*, (5):38, 1998.

Abstract: It is shown, with the example of the experimentally known Adler function, that there is no matching in the intermediate region between the two asymptotic regimes described by perturbative QCD (for the very short-distances) and by chiral perturbation theory (for the very long-distances). We then propose to consider an approximation of

large-N-c QCD which consists in restricting the hadronic spectrum in the channels with J(P) quantum numbers 0(-), 1(-), 0(+) and 1(+) to the lightest state and to treat the rest of the narrow states as a perturbative QCD continuum; the onset of this continuum being fixed by consistency constraints from the operator product expansion. We show how to construct the low-energy effective Lagrangian which describes this approximation. The number of free parameters in the resulting effective Lagrangian can be reduced, in the chiral limit where the light quark masses are set to zero, to just one mass scale and one dimensionless constant to all orders in chiral perturbation theory. A comparison of the corresponding predictions, to O(p(4)) in the chiral expansion, with the phenomenologically known couplings is also made.

[492] T. M. Perrine, R. K. Chaudhuri, and K. F. Freed. Quadratic Pade approximants and the intruder state problem of multireference perturbation methods. *Int. J. Quantum Chem.*, 105(1):18–33, 2005.

Abstract: Simple and quadratic Pade resummation methods are applied to high-order series from multireference many-body perturbation theory (MR-MBPT) calculations using various partitioning schemes (Moller-Plesset, Epstein-Nesbet, and forced degeneracy) to determine their efficacy in resumming slowly convergent or divergent series. The calculations are performed for the ground and low-lying excited states of (i) CH2, (ii) BeH2, at three geometries, and (iii) Be, for which full configuration interaction (0) calculations are available for comparison. The 49 perturbation series that are analyzed include those with oscillatory and monotonic divergence and convergence, including divergences that arise from either frontdoor or backdoor intruder states. Both the simple and quadratic Pade approximations are found to speed the convergence of slowly convergent or divergent series. However, the quadratic Pade method generally outperforms the simple Pade resummation. (c) 2005 Wiley Periodicals, Inc.

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Abstract: We derive consistency conditions which constrain the possible form of the strong couplings of the excited baryons to the pions. The consistency conditions follow from requiring the pion-excited baryon scattering amplitudes to satisfy the large-N-c Witten counting rules and are analogous to consistency conditions used by Dashen, Jenkins, and Manohar and others for s-wave baryons. The consistency conditions are explicitly solved, giving the most general allowed form of the strong vertices for excited baryons in the large-N-c limit. We show that the solutions to the large-N-c consistency conditions coincide with the predictions of the non-relativistic quark model for these states, extending the results previously obtained for the s-wave baryons. The 1/N-c corrections to these predictions are studied in the quark model with arbitrary number of colors N-c.

[494] P. V. Pobylitsa. Baryon distribution amplitude: Large-N-c factorization, spin-flavor symmetry and soft-pion theorem. *Phys. Rev. D*, 72(1):20, 2005.

Abstract: The 1/N-c expansion for the baryon distribution amplitude is constructed in terms of a specially designed generating functional. At large N-c this functional shows exponential behavior. The exponential factor is universal for all low-lying baryons and baryon-meson scattering states. Simple factorization properties are established for the preexponential term. This factorization agrees with the large-N-c contracted SU(2N(f)) spin-flavor symmetry. The consistency of the factorization with the soft-pion theorem for the baryon distribution amplitude is explicitly checked. A relation between the generating functionals for the distribution amplitudes of the nucleon and the Delta resonance is derived.

[495] V. S. Popov. ON THE THEORY OF THE ABOVE-THE-BARRIER STARK RESONANCES. *Phys. Lett. A*, 173(1):63–68, 1993.

Abstract: A semiclassical 1/n-expansion for the Stark effect in a strong electric field epsilon is considered, which gives simple analytic formulae determining the atomic widths GAMMA(n)(epsilon).

It has been shown that there is a range in which the widths of the Stark resonances depend almost linearly on the applied electric field, which agrees with numerical calculations.

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Abstract: Calculations are made of the energy and angular distributions of photoelectrons during tunneling ionization of an atom or an ion under the action of high-power laser radiation (for all values of the Keldysh parameter gamma). Cases of linear, circular, and elliptic polarizations of the electromagnetic wave are considered. The probability of above-barrier ionization of hydrogen atoms in a low-frequency (gamma much less than 1) laser field is calculated. Formulas are given for the momentum spectrum of the electrons when an atomic level is ionized by a general type of alternating electric field (for the case of linear polarization). An analysis is made of tunneling interference in the energy spectrum of the photoelectrons. Analytic approximations are discussed for the asymptotic coefficient C-kappa of the atomic wave function at infinity (for s-wave electrons). (C) 2000 MAIK "Nauka/Interperiodica".

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Abstract: Using the 1/n-expansion, we obtain analytic formulae for the bound state radial wave functions, including its asymptotic coefficients at r –¿ 0 and r –¿ infinity, for an arbitrary smooth potential V(r). The formulae are asymptotically exact in the limit n( )r–¿ infinity (n = n(r) + l + 1 is the principal quantum number and the expansion parameter is 1/n). Comparison with exact solutions and numerical calculations for the power-law and short-range potentials show that the applicability region of these formulae is usually prolonged up to small quantum numbers, n similar to 1. With growing n(r), the accuracy of the formulae decreases, but the WKB method becomes applicable in this case.

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Abstract: Using the "imaginary time" method we have calculated (in the quasiclassical approximation) the probability of ionization of the atomic s-state in static electric and magnetic fields. The Coulomb interaction between the emitted electron and the atomic remainder is taken into account. The results obtained are valid for external fields E and H which are smaller than characteristic atomic fields. The case of mutually orthogonal fields (the Lorentz ionization) is carefully studied. (C) 1997 Elsevier Science B.V.

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Abstract: A semiclassical theory is developed for the ionization of atoms and negative ions in constant, uniform electric and magnetic fields, including the Coulomb interaction between the electron and the atomic core during tunneling. The case of crossed fields (Lorentz ionization) is examined specially, as well as the limit of a strong magnetic field. Analytic equations are derived for arbitrary fields E and H that are weak compared to the characteristic intraatomic fields. The major results of this paper are obtained using the "imaginary time" method (ITM), in which tunneling is described using the classical equations of motion but with purely imaginary "time." The possibility of generalizing the ITM to the relativistic case, as well as to states with nonzero angular momentum, is pointed out. (C) 1998 American Institute of Physics. [S1063-7761(98)00405-3].

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Abstract: A perturbation-theory formula is derived for the energies of quasistationary states (resonances) in the semiclassical approximation. This formula is valid for resonances either below or

above the barrier. It is illustrated for several potentials, for which a comparison can be made with exact solutions.

[501] V. S. Popov, V. D. Mur, and A. V. Sergeev. 1/N-EXPANSION AND SCALING FOR THE STARK-EFFECT IN RYDBERG ATOMS. *Phys. Lett. A*, 149(9):425–430, 1990.

[502] V. S. Popov, V. D. Mur, and A. V. Sergeev. QUANTIZATION RULES FOR QUASI-STATIONARY STATES. *Phys. Lett. A*, 157(4-5):185–191, 1991.

Abstract: The modification of the Bohr-Sommerfeld quantization rules, which is due to the barrier penetrability, is found. The equation obtained is valid for an arbitrary analytical potential U(x), obeying the quasiclassical conditions. It determines both the position E(r) and the width-GAMMA of the quasistationary state. A generalization of the Gamow formula for multidimensional systems with separable coordinates is derived. A comparison with exactly solvable models as well as with numerical solutions of the Schrodinger equation for the Stark problem is performed.

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Abstract: A correction to the quasiclassical quantization rules is found which is connected with the taking into account of the barrier penetration. The equation derived defines both the position E(r) and width GAMMA of the quasistationary level. The results are compared with numerical solutions of the Schrodinger equation and with the exactly soluble models. A generalization of the Gamov formula for systems with separated variables is obtained. The possibility of extending the results to the relativistic case is discussed.

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Abstract: Exact values of the critical field E(c) are calculated for a hydrogen atom, including the

case of the ground state. The widths GAMMA(n) of Stark resonances at E = E(c) are also calculated. In the above-barrier region (E ¿ E(c)) the widths GAMMA(n)(E) are essentially linear functions of the electric field strength.

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Abstract: The asymptotic form of higher orders of the 1/n expansion in quantum mechanics is factorial. The Yukawa potential and the hydrogen atom in electric and magnetic fields are discussed.

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Abstract: The asymptotics of large orders of the 1/n expansion in quantum mechanics has been found. It is shown that the coefficients epsilon(k) grow as k!a(k) With k–¿infinity, and the dependence of the parameter a on the coupling constant is investigated.

[510] V. S. Popov and A. V. Sergeev. ASYMPTOTICS OF THE HIGHEST ORDERS OF 1-N-DECOMPOSITION FOR MULTIDIMENSIONAL PROBLEMS. *Zhurnal Eksperimentalnoi Teor. Fiz.*, 105(3):568–591, 1994.

[511] V. S. Popov and A. V. Sergeev. LARGE ORDERS OF 1/N-EXPANSION FOR MULTIDIMENSIONAL PROBLEMS. *Phys. Lett. A*, 193(2):165–172, 1994.

Abstract: The asymptotics of large orders of the 1/n-expansion is investigated for multidimensional problems of quantum mechanics and atomic physics, including those with separable variables (the hydrogen molecular ion H-2(+)), and those where separation of variables is impossible (a hydrogen atom in electric and magnetic fields). It is shown that the parameters of the asymptotics can be found by means of calculating sub-barrier trajectories with the help of the "imaginary time" method, as well as by solution of the eikonal equation.

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Abstract: The ionization probability of an atomic s state under the action of static electric and magnetic fields is calculated taking into account the Coulomb interaction between the escaping electron and the atomic core. The structure of the perturbation series for the energy of the level is investigated and the asymptotic behavior of the higher orders of the perturbation theory is found. (C) 1996 American Institute of Physics.

[513] V. S. Popov and A. V. Sergeev. Ionization of atoms in weak fields and the asymptotic behavior of higher-order perturbation theory. *J. Exp. Theor. Phys.*, 86(6):1122–1126, 1998.

Abstract: Using the imaginary time method, we study the structure of the perturbation series for the hydrogen atom in electric E and magnetic H fields. It is shown that there is a "critical" value of the ratio gamma = H/E at which the perturbation series for the ground state changes from having a fixed sign (for gamma ¡ gamma(c)) to having a variable sign (for gamma ¿ gamma(c)). This conclusion is confirmed by direct higher-order perturbation calculations. The change in the asymptotic regime is explained by competition among the contributions of the various complex trajectories that describe the subbarrier motion of the electrons. Here the parameter gamma(c) depends on the angle theta between the electric and magnetic fields. (C) 1998 American Institute of Physics.

[514] V. S. Popov, A. V. Sergeev, V. D. Mur, and A. V. Shcheblykin. ON THE ASYMPTOTICS OF HIGH-ORDER TERMS OF THE 1/N EXPANSION. *Phys. Atom. Nuclei*, 57(6):1057–1065, 1994.

Abstract: Analytic and numerical methods for determining the asymptotics of high-order terms of the 1/n expansion in quantum-mechanical problems are developed. It is shown that this asymptotics is always of the factorial type. The dependence of parameters of the asymptotics on the form of the potential and on the coupling constant is especially analyzed in the vicinity of the point of collision of classical solutions.

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Abstract: The asymptotic behaviour of large orders in 1/n-expansion is found for the problems of quantum mechanics. The coefficients of 1/n-expansion epsilon(k) are shown to increase as a factorial k! a(k) at k–¿infinity. The a-parameter vs coupling constant dependence is studied. The analytic formulas obtained agree with the numerical calculations. The structure of 1/n-expansion is considered in detail for the Yukawa, Hulten and funnel potentials, as well as for the Stark effect in hydrogen and for the molecular ion H.

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the increase of computational cost with the number of degrees of freedom is analysed. Quadratic Pade approximants of the perturbation expansions are rapidly convergent, and directly yield complex numbers for resonance eigenvalues. For a 3-mode system, results are obtained within partial separability assumptions, with a pair of modes left coupled. Large-order perturbation theory with partial separability is suggested as an alternative to low-order exact perturbation theory.

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the rate of convergence of the diagonal staircase approximant sequence increases with the degree. Different branches of the approximants converge to different branches of the function. The success of the high-degree approximants is attributed to their ability to model the function on multiple sheets of the Riemann surface and to reproduce the correct singularity structure in the limit of large perturbation parameter. An efficient recursive algorithm for computing the diagonal approximant sequence is presented.

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used the true Hamiltonian without the approximation of a finite basis set. In practice, the singularity structure depends strongly on the choice of basis set. Standard basis sets cannot model dissociation to an electron cluster, but if the basis includes diffuse functions then it can model another critical point corresponding to complete dissociation of all the valence electrons. This point is farther from the origin of the z plane than is the critical point for the electron cluster, but it is still close enough to cause divergence of the perturbation series. For the hydrogen fluoride molecule a critical point is present even without diffuse functions. The basis functions centered on the H atom are far enough from the F atom to model the escape of electrons away from the fluorine end of the molecule. For the Ar atom a critical point for a one-electron ionization, which was not previously predicted, seems to be present at a positive value of the perturbation parameter. Implications of the existence of critical points for quantum-chemical applications are discussed. (C) 2005 American Institute of Physics.

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large-D limit is completely analogous to the standard phase transitions and critical phenomena in statistical mechanics. Electronic structure at the large-D limit exhibits critical points with mean field critical exponents (beta = 1/2, alpha = 0(dis), delta = 3, and gamma = 1). The complete mapping is presented for the Hartree-Fock two-electron atom in weak electric field and the two Coulomb center problems.

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Abstract: Is chemistry a science without a territory? I argue that "chemical bonding" has been a traditional chemical territory ever since the chemical community amalgamated in the seventeenth century, and even before. The modem charter of this territory is Gilbert Newton Lewis, who started the "electronic structure revolution in chemistry." As a tribute to Lewis, I describe here three of his key papers from the years 1913, 1916, and 1923, and analyze them. Lewis has defined the quantum unit, the "electron pair bond," for construction of a chemical universe, and in so doing he charted a vast chemical territory and affected most profoundly the mental map of chemistry for generations ahead. Nevertheless, not all is known about the chemical bond" the chemical territory is still teaming with new and exciting problems of in new materials, nanoparticles, quantum dots, metalloenzymes, bonding at surface-vapor interfaces, and so on and so forth. (C) 2006 Wiley Periodicals, Inc.

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of the ground state energy as a function of the coupling parameters in the Hamiltonian. In this approach, the finite size corresponds to the number of elements in a complete basis set used to expand the exact eigenfunction of a given molecular Hamiltonian. To illustrate this approach, we give detailed calculations for systems of one electron and two nuclear centres, Z(+)e(+)Z(+). Within the Born-Oppenheimer approximation, there is no critical point, but without the approximation the system exhibits a critical point at Z = Z(c) = 1.228 279 when the nuclear charge, Z, varies. We show also that the dissociation occurs in a first- order phase transition and calculate the various related critical exponents. The possibility of generalizing this approach to larger molecular systems using Gaussian basis sets is discussed.

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h-expansion allows to apply the quantization condition for involving thte wavefunction nodes, what results in the simple recursive relations. In the example of the funnel - shaped potential the calculations are given for the first ten coefficients in the 1/n-expansion scheme for energy with the various values of the orbital and radial quantum numbers.

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Abstract: Planck's constant h is a fundamental physical parameter that establishes the scale of quantum phenomena. Bound-state energy eigenvalues for several well-known systems (e.g., harmonic oscillators, Morse oscillators, and square-well potentials) are formally analytic functions of

h in the neighborhood of h = 0, and any "physical" state can be reached in principle by analytic continuation. This paper explores the possibility of developing power series in h for atomic and molecular energies, with analytic continuation to access physical states. For this purpose it is necessary to modify Coulomb interactions at short range to keep the potential bounded below. As a result the h expansions appear naturally as Rayleigh-Schrodinger perturbation series in anharmonicity about collective harmonic-oscillator states. Physical eigenvalues emerge in the strong-coupling limit, and Pade approximants seem to be analytical tools well suited for entering the regime. Some basic implementation details are presented for application to the modified hydrogen atom, the two-electron isoelectronic atomic sequence, and many-electron atoms.

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space. In the spirit of the electrostatic theorem, we introduce the concept of antibinding and binding regions to visualize chemical binding. In addition, criteria are developed to determine the ionic or covalent character of the bond. The computational effort requires only the minimization of a multidimensional potential surface at the infinite dimensional limit, yet detailed information can be extracted. Numerical examples are given for the one-electron diatomic molecule.

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Abstract: It is generally believed that the old quantum theory, as presented by Niels Bohr in 1913, fails when applied to few electron systems, such as the H-2 molecule. Here we review recent developments of the Bohr model that connect it with dimensional scaling procedures adapted from quantum chromodynamics. This approach treats

electrons as point particles whose positions are determined by optimizing an algebraic energy function derived from the large-dimension limit of the Schrodinger equation. The calculations required are simple yet yield useful accuracy for molecular potential curves and bring out appealing heuristic aspects. We first examine the ground electronic states of H-2, HeH, He-2, LiH, BeH and Li-2. Even a rudimentary Bohr model, employing interpolation between large and small internuclear distances, gives good agreement with potential curves obtained from conventional quantum mechanics. An amended Bohr version, augmented by constraints derived from Heitler-London or Hund-Mulliken results, dispenses with interpolation and gives substantial improvement for H-2 and H-3. The relation to D-scaling is emphasized. A key factor is the angular dependence of the Jacobian volume element, which competes with interelectron repulsion. Another version, incorporating principal quantum numbers in the D-scaling transformation, extends the Bohr model to excited S states of multielectron atoms. We also discuss kindred Bohr-style applications of D-scaling to the H atom subjected to superstrong magnetic fields or to atomic anions subjected to high frequency, superintense laser fields. In conclusion, we note correspondences to the prequantum bonding models of Lewis and Langmuir and to the later resonance theory of Pauling, and discuss prospects for joining D-scaling with other methods to extend its utility and scope.

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Abstract: It is generally believed that the old quantum theory, as presented by Niels Bohr in 1913, fails when applied to few electron systems, such as the H-2 molecule. Here, we find previously undescribed solutions within the Bohr theory that describe the potential energy curve for the lowest singlet and triplet states of H-2 about as well as the early wave mechanical treatment of Heitler and London. We also develop an interpolation scheme that substantially improves the agreement with the exact ground-state potential curve of H-2 and provides a good description of more complicated molecules such as LiH, Li-2, BeH, and He-2.

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Abstract: The Green's function for the Helmholtz differential operator del(2) +lambda(lambda+N - 1) on the N-dimensional (with N ¿= 1) hyperspherical surface S-N of unit radius is investigated. Its closed form is shown to be G((N))(lambda; n, n') = pi/(N - 1)S-N sin (pi lambda) C-lambda(((N-1)/2)) (-n (.) n'), where S-N is the area of S-N, C-lambda((alpha)). (x) is the Gegenbauer function of the first kind, while n and n' are radius vectors, with respect to the centre of S-N, of the observation and source points, respectively. The Green's function G((N))(lambda; n, n') fails to exist whenever. is such that it holds that lambda(lambda + N - 1) = L(L + N - 1), with L is an element of N. For these exceptional cases, the generalized (known also as 'modified' or 'reduced') Green's function G(L)((N)) (n, n') is considered. It is shown that G(L)((N)) (n, n') may be expressed compactly in terms of the Gegenbauer polynomial C-L(((N- 1)/ 2)) (n (.) n') and the derivative [6C(lambda)((N-1)/2)) (-n (.) n')/6 lambda](lambda=L). Explicit expressions for the derivatives [6C(lambda)((n)) (x)/6 lambda](lambda=L) and [6C(lambda)((n+1/2)) (x)/6 lambda](lambda=L), with n is an element of N, are found and used to transform the functions G(L)((2n+1)) (n, n') and G(L)((2n+2)) (n, n') to potentially more useful forms.

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that for a given field strength DPT would be effective only when m is large. However, the field-strength expression used in DPT is scaled as a function of m, so it is not obvious a priori whether the effectiveness of DPT diminishes when m is significantly reduced for a given physical field strength. It is shown that for many states of diamagnetic hydrogen DPT can still produce strongly convergent and accurate energy values when m is small, even when m = 0 (delta = 1/2). For those regions where even Pade summation failed to converge adequately, a technique is presented based on economized rational approximants.

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Fermi pseudopotentials are derived. The Green's functions, the propagators, and the exact solutions of the Lippmann-Schwinger equations are derived in explicit forms. In odd dimensions d = 1 and 3 the Fermi pseudopotentials can be applied to describe multiphoton processes of atoms and molecules with very short-range interactions. In even dimension d = 2 the Fermi pseudopotential can be applied to describe tunneling from laser-driven quantum wells. Physical applications involving higher d are also possible.

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