Final distribution

including quasiclassical correction $\sim \hbar^2$

$$\rho^{(\mathrm{F})}(\vec{q},\vec{p}) = \exp\left(-(H^{(\mathrm{F})} - E) f_1 / 3 f_2 - 2\hbar^2 f_1^3 / 27 f_2^2\right)$$
$$\times \mathrm{Ai}\left(\alpha (H^{(\mathrm{F})} - E + \hbar^2 f_1^2 / f_2)\right)$$

$$\alpha = \left(3\hbar^2 f_2\right)^{-1/3}$$

$$f_1 = \sum_i V_{ii}^{(\mathrm{F})}{}'', \quad f_2 = \sum_i \left(V_i^{(\mathrm{F})}\right)^2 + \sum_{i,j} V_{ij}^{(\mathrm{F})}{}'' p_i p_j$$

Alternative formula

$$\rho_E^{(\mathrm{F})}(\vec{q},\vec{p}) = \delta\left(E - H^{(\mathrm{F})}(\vec{q},\vec{p})\right)$$
$$+ \hbar^2 \left[\tfrac{1}{8} f_1(\vec{q},\vec{p}) \delta''\left(E - H^{(\mathrm{F})}(\vec{q},\vec{p})\right) + \tfrac{1}{24} f_2(\vec{q},\vec{p}) \delta'''\left(E - H^{(\mathrm{F})}(\vec{q},\vec{p})\right) \right]$$

Transition rate

$$I_1 = \exp\left(-\frac{\lambda^3}{3\hbar^3} f_2^*\right) I_0$$

Logarithm of the rate

$$I = \exp\left(-\tfrac{2}{\hbar} w\right), \quad w = w_0 + \hbar^2 w_1.$$
$$w_0 = w^* - \tfrac{\hbar}{2} \ln\left(\left(\pi\hbar H_1^2 \,\mathrm{Det}\,\mathbf{F}\right)^{-1/2} f_0^*\right)$$
$$w_1 = \tfrac{1}{6} \hbar^{-2} \lambda^3 f_2^*$$

Condition of convergence

$$\left|\tfrac{1}{6} \lambda^3 f_2^*\right| << W^*$$