Testing formulas for decay rates

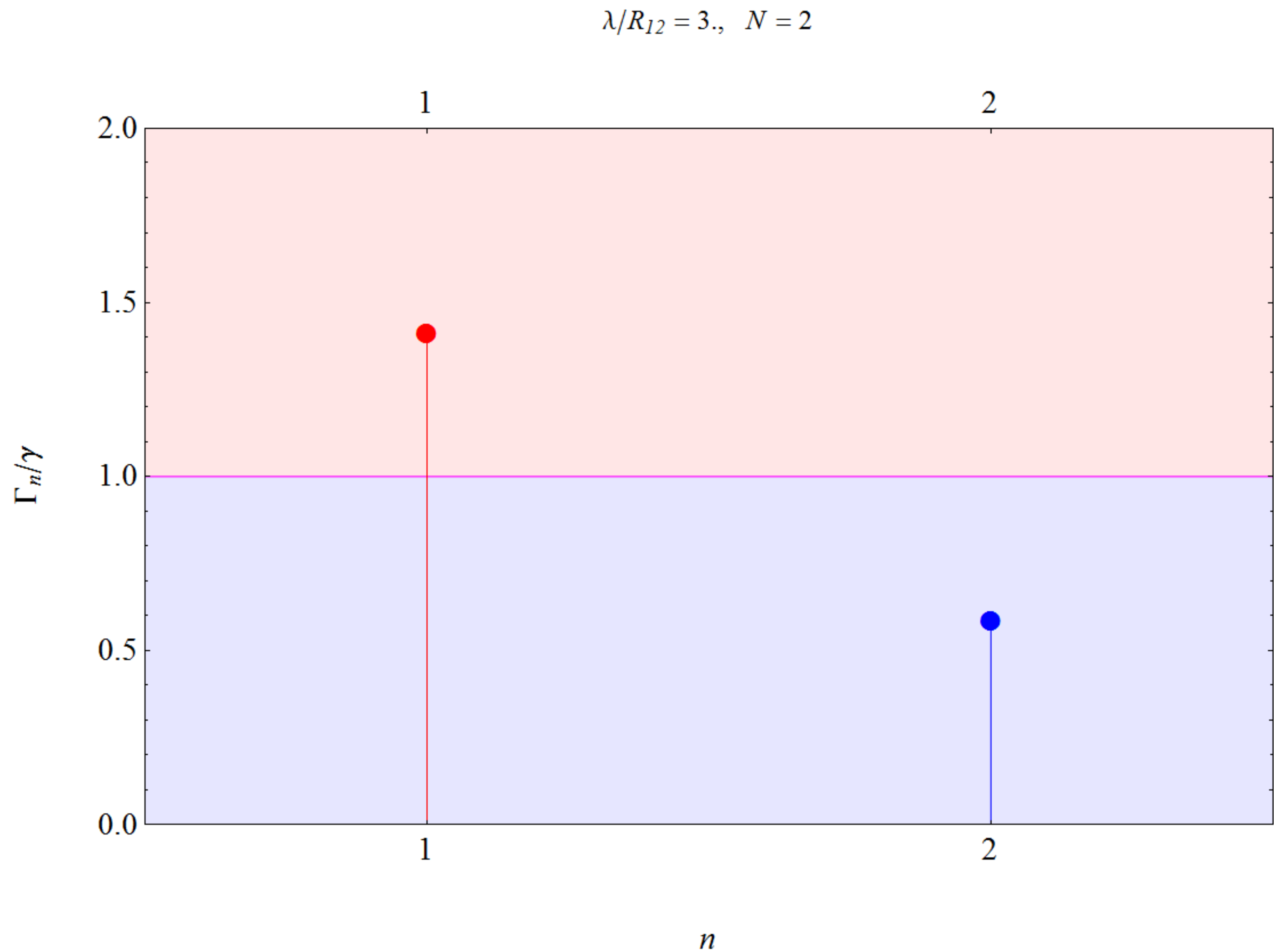
Equations for decay rates

$$\left\{ \Gamma_i^{(N)} /_{i=1,2,\ldots,N} \right\} = \gamma \times \text{eigenvalues}(\mathbf{F})$$

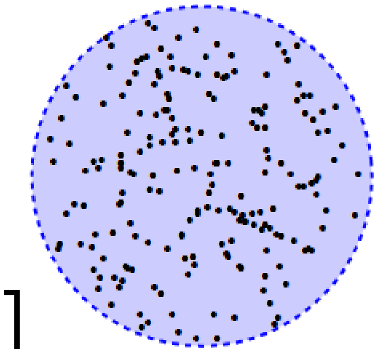
$$F_{ij} = \begin{cases} 1, & i = j \\ \dfrac{\sin\left(k_0 \left|\vec{r}_i - \vec{r}_j\right|\right)}{k_0 \left|\vec{r}_i - \vec{r}_j\right|}, & i \neq j \end{cases}$$

$$N = 1: \quad \Gamma_1^{(N)} = \gamma$$

$$N = 2: \quad \Gamma_1^{(N)} = \gamma(1 + F_{12}), \quad \Gamma_2^{(N)} = \gamma(1 - F_{12})$$



N atoms inside a sphere: theoretical prediction



$$\Gamma_i^{(N)} \approx \frac{3N}{2}\left[j_n^2(k_0 r) - j_{n-1}(k_0 r)\, j_{n+1}(k_0 r) \right],$$

where $n = \left\{ i^{1/2} \right\} - 1$

Coherent Emission of a Photon by Many Atoms

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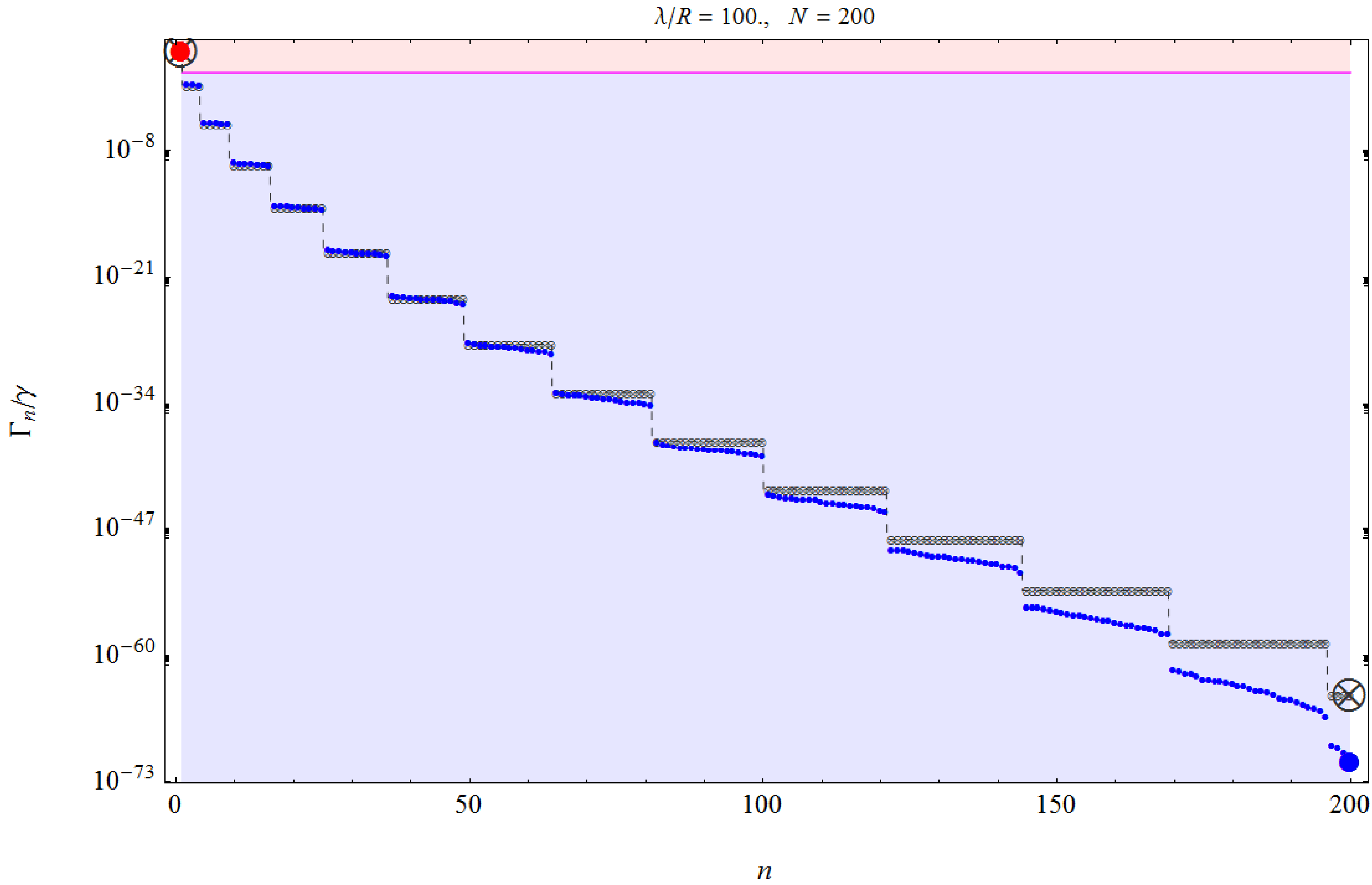
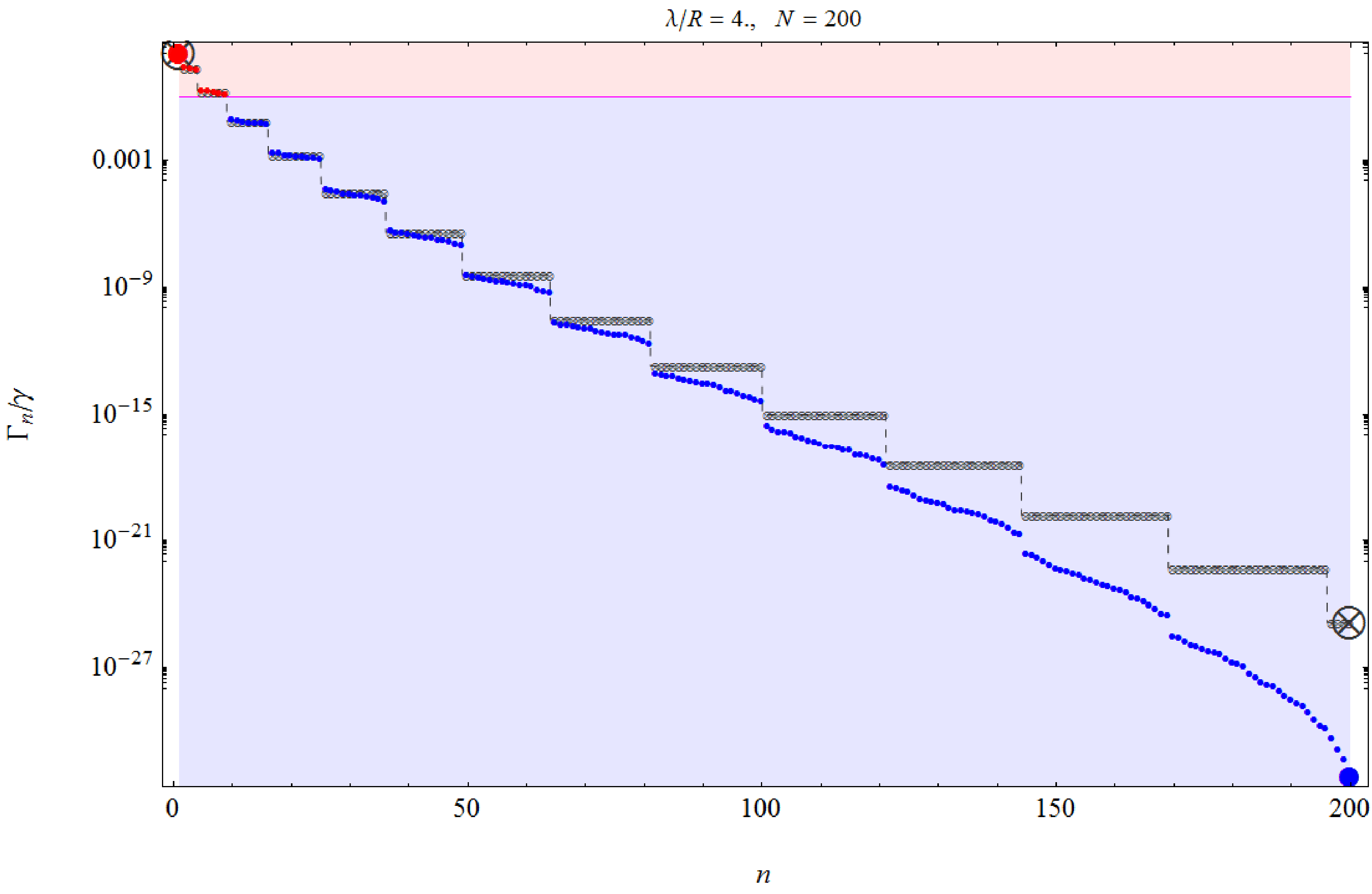
The theory of Weisskopf and Wigner[1] of the natural line width is extended to a case where many atoms at given positions interact with one common quantized radiation field. At time $t=0$, one quantum of radiation energy is stored by the atoms, which however does not mean necessarily that exactly one of the atoms must be excited. We study the process of the creation of a photon in dependence on the positions of the atoms and on the state of the system of atoms at $t=0$. The dimensions of the emitted wave train and the decay time of this excited atomic state depend sensitively on these parameters.

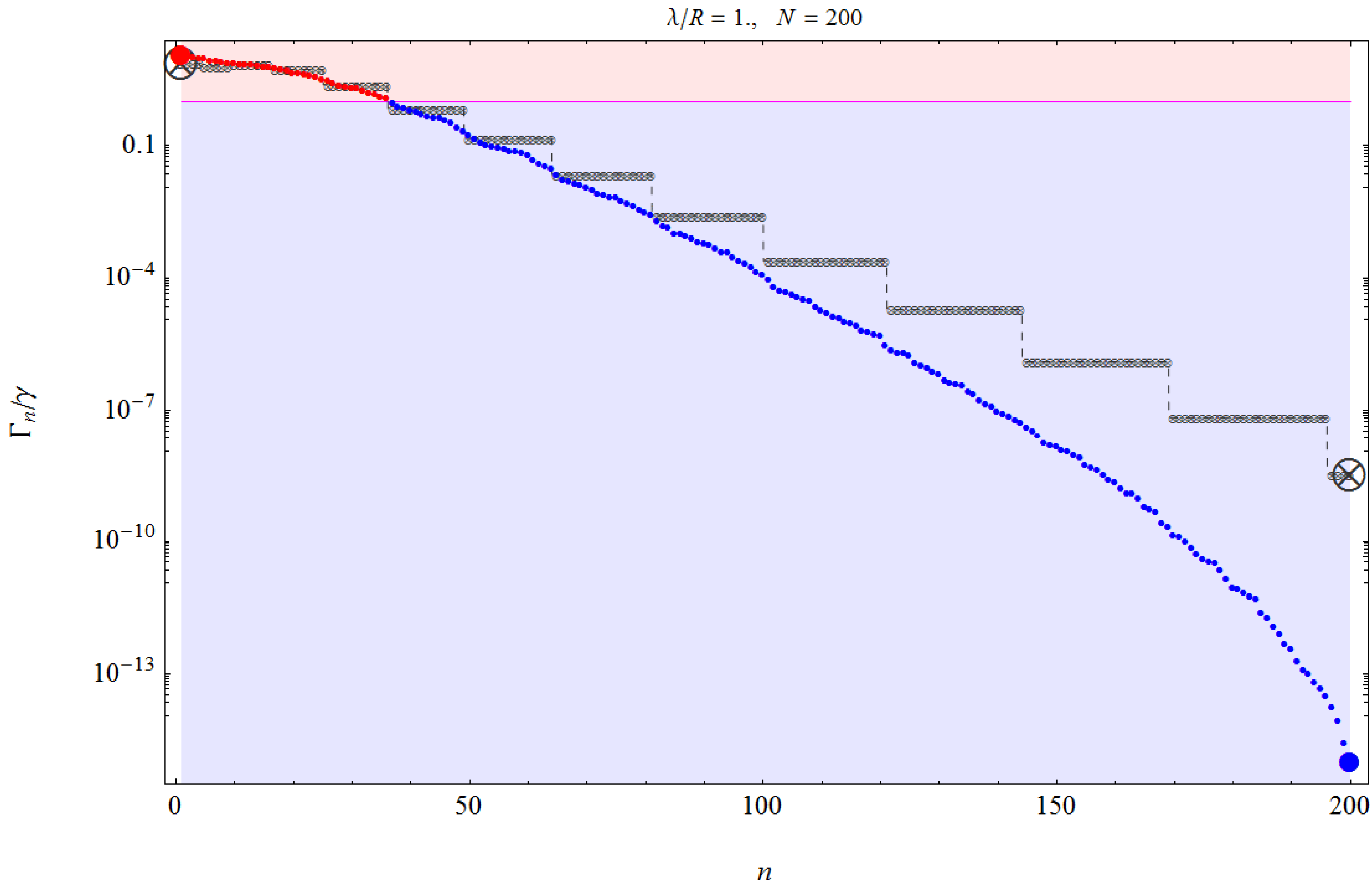
Large wavelength



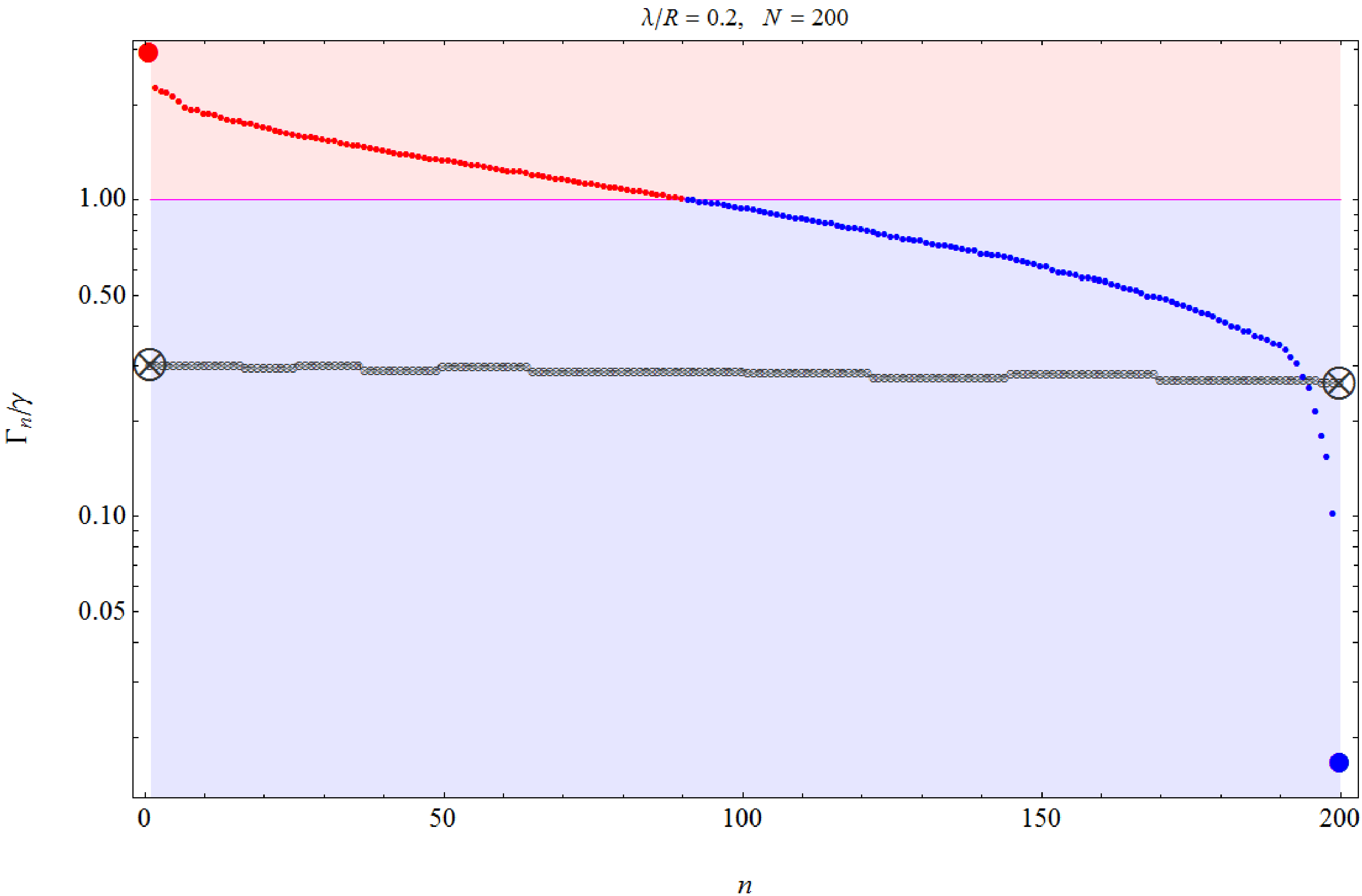
Wavelength 4 times larger than size of sphere



Wavelength equal to radius of sphere

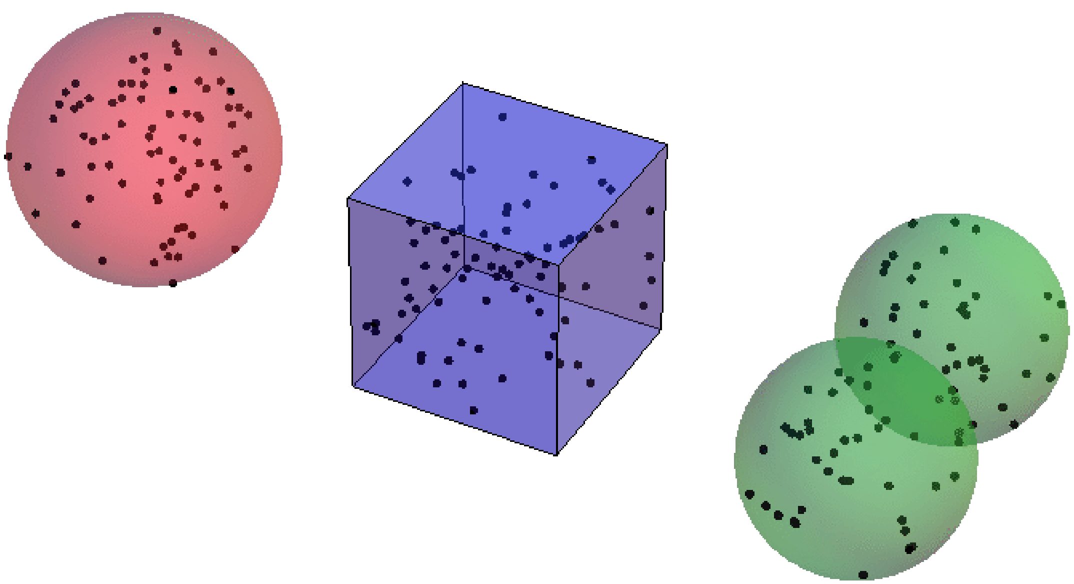


Small wavelength

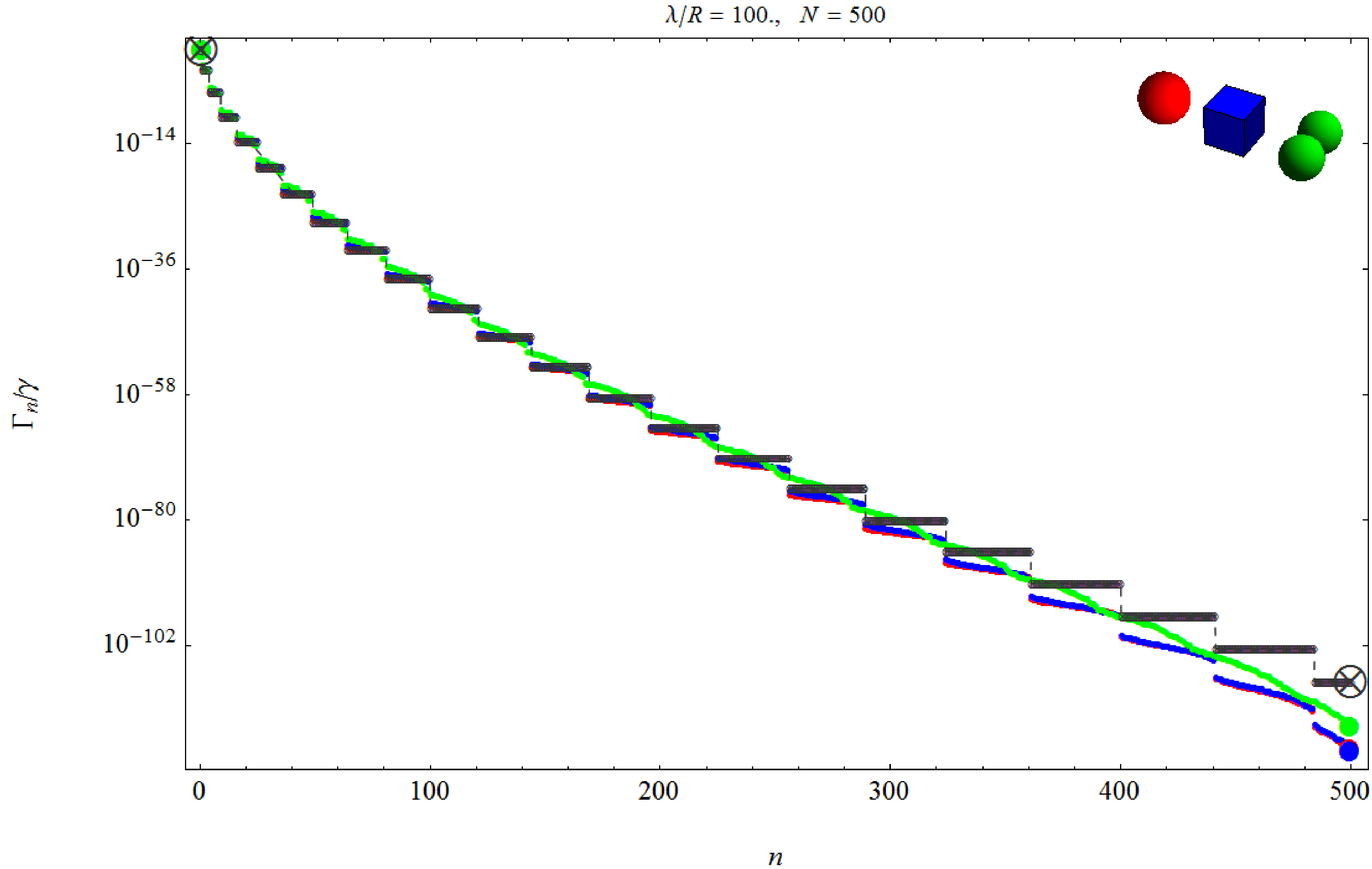


Dependence on shape

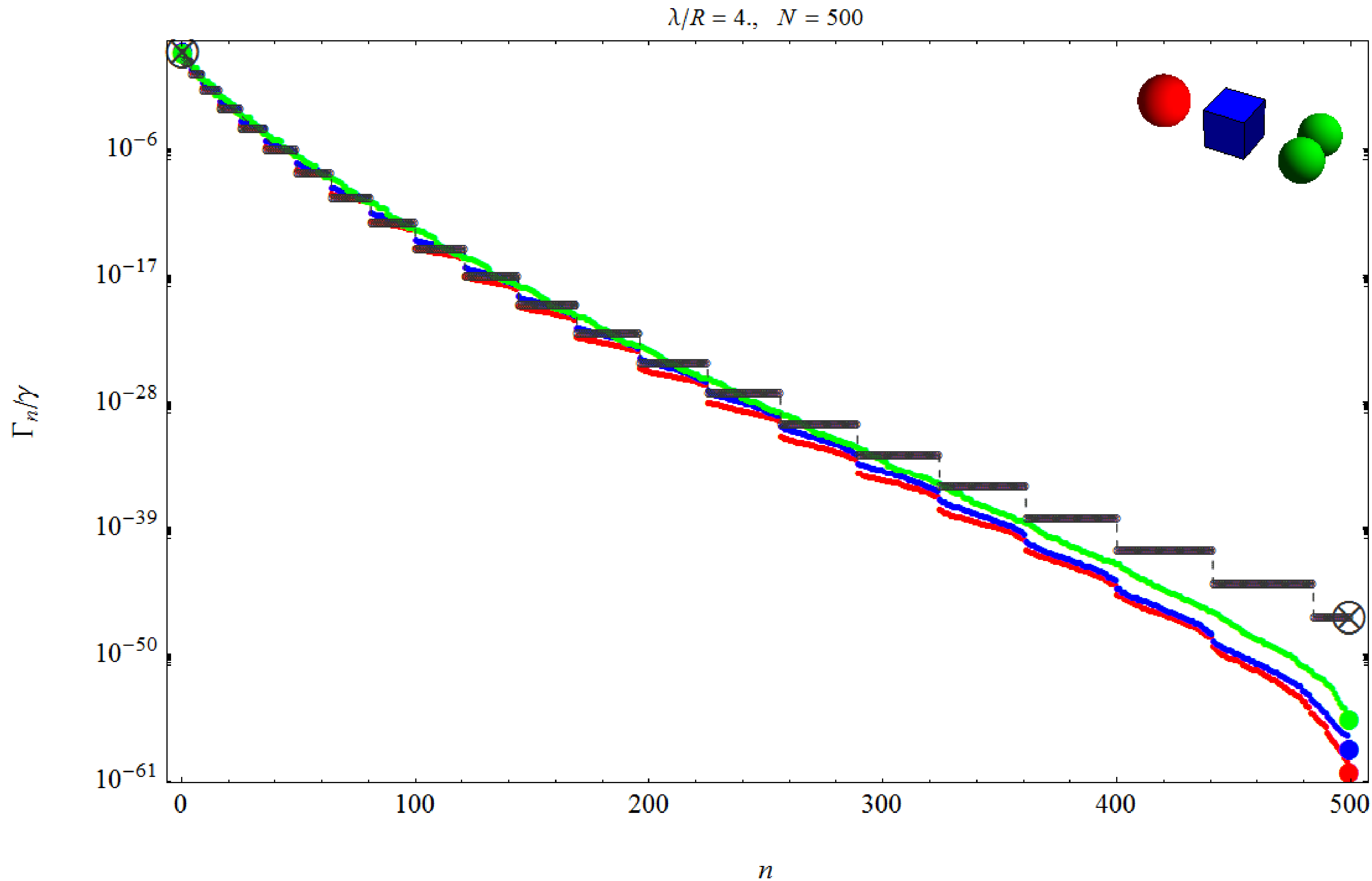
of linear dimensions large in comparison with $\lambda/2$. This means that $\mathscr{C}_L{}^M(x)$ is still an approximate eigenstate for any large, three-dimensional region of a shape not too much different from a sphere, provided, of course, that the radiation law is isotropic. As for the latter, however, the



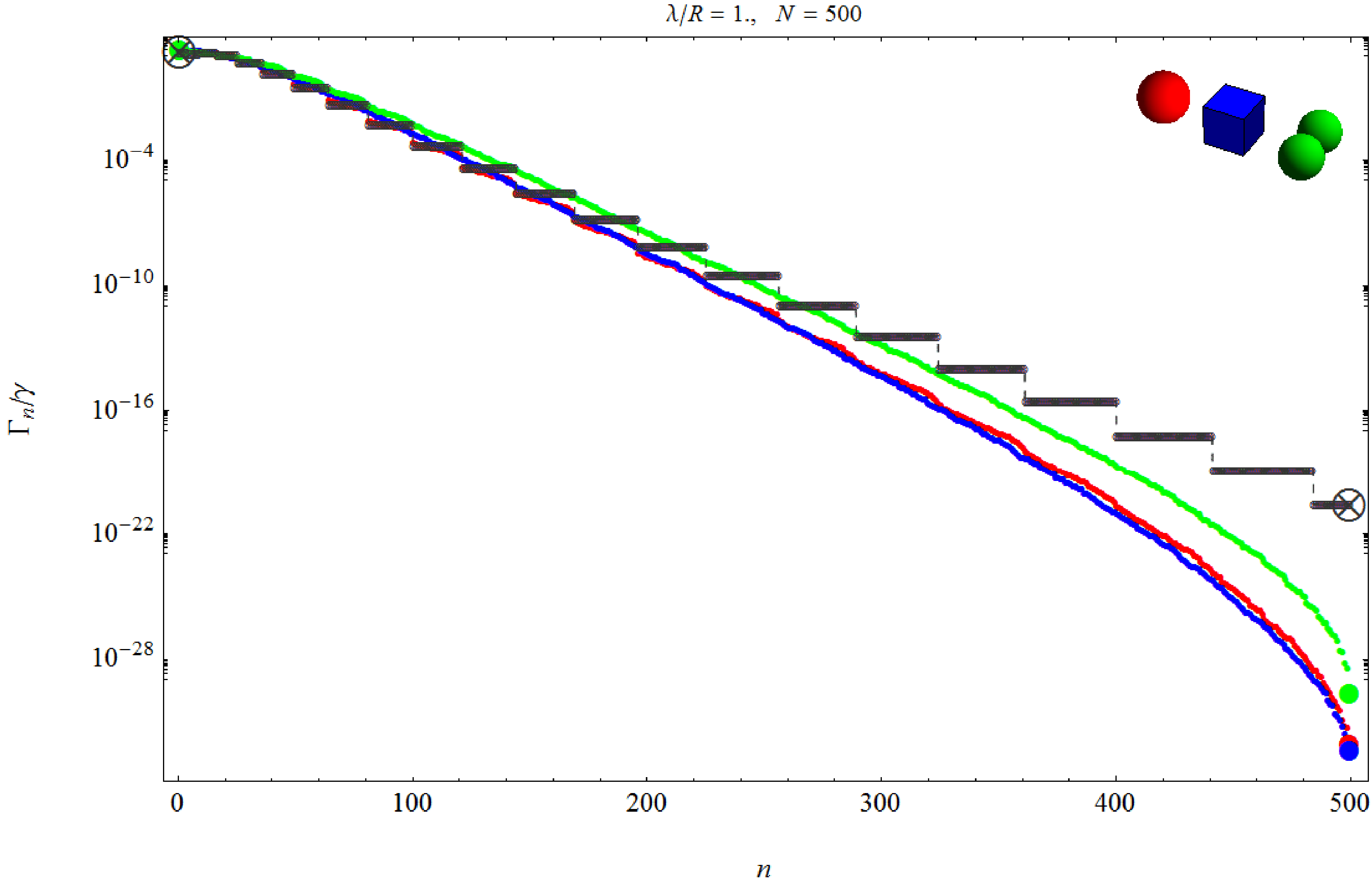
Dependence on shape, large wavelength



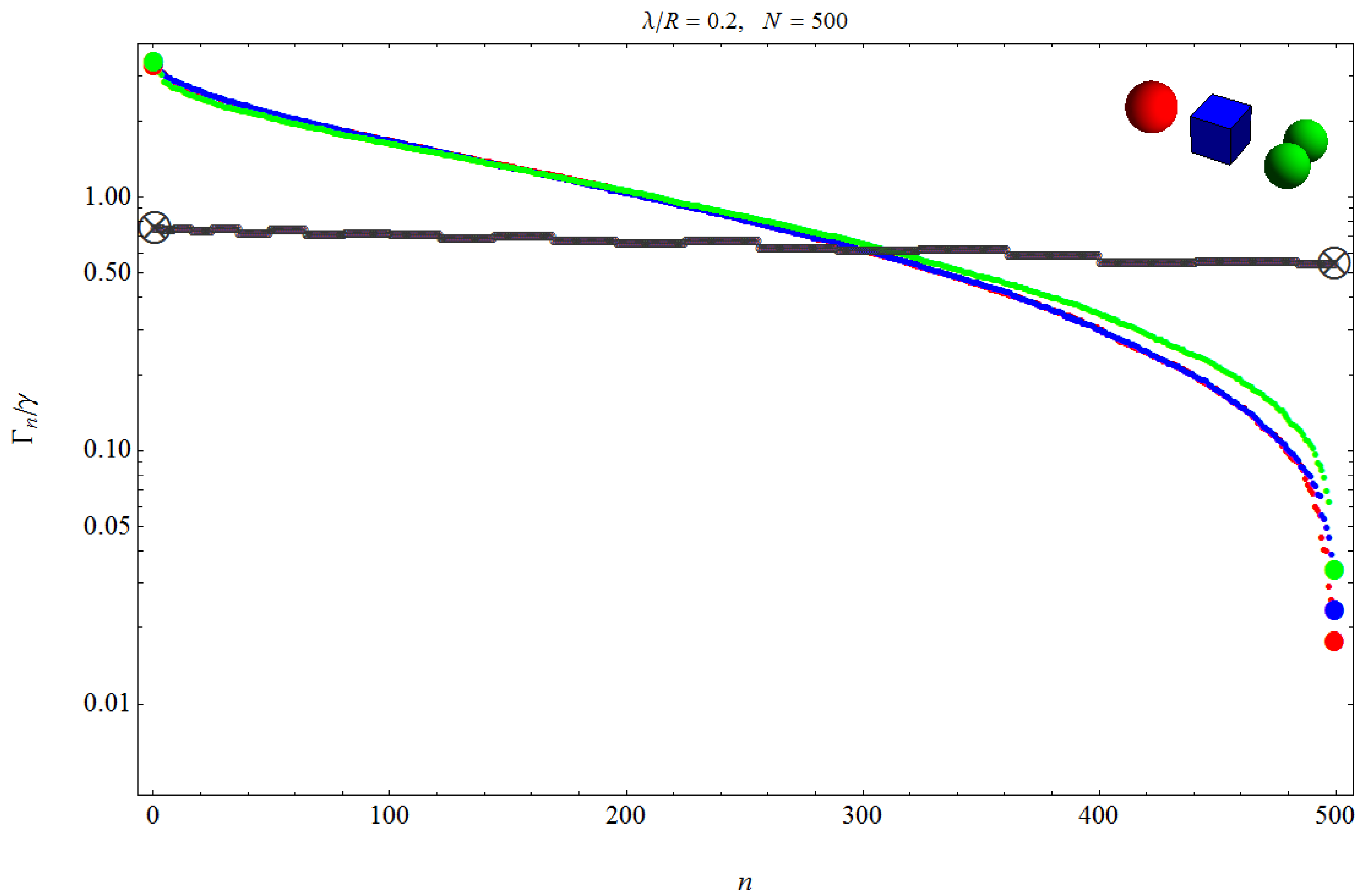
Dependence on shape, wavelength larger than size



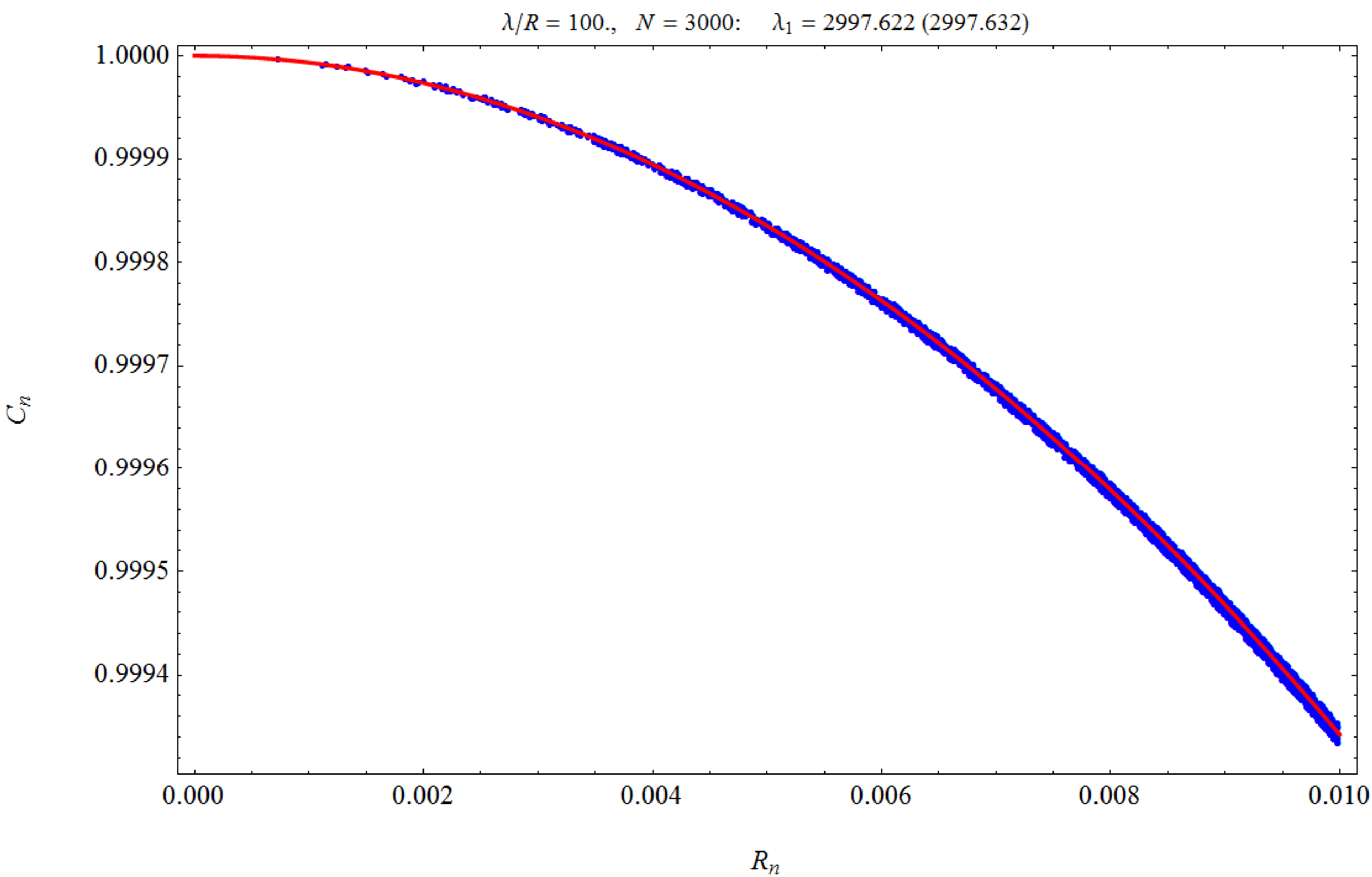
Dependence on shape, wavelength comparable with size



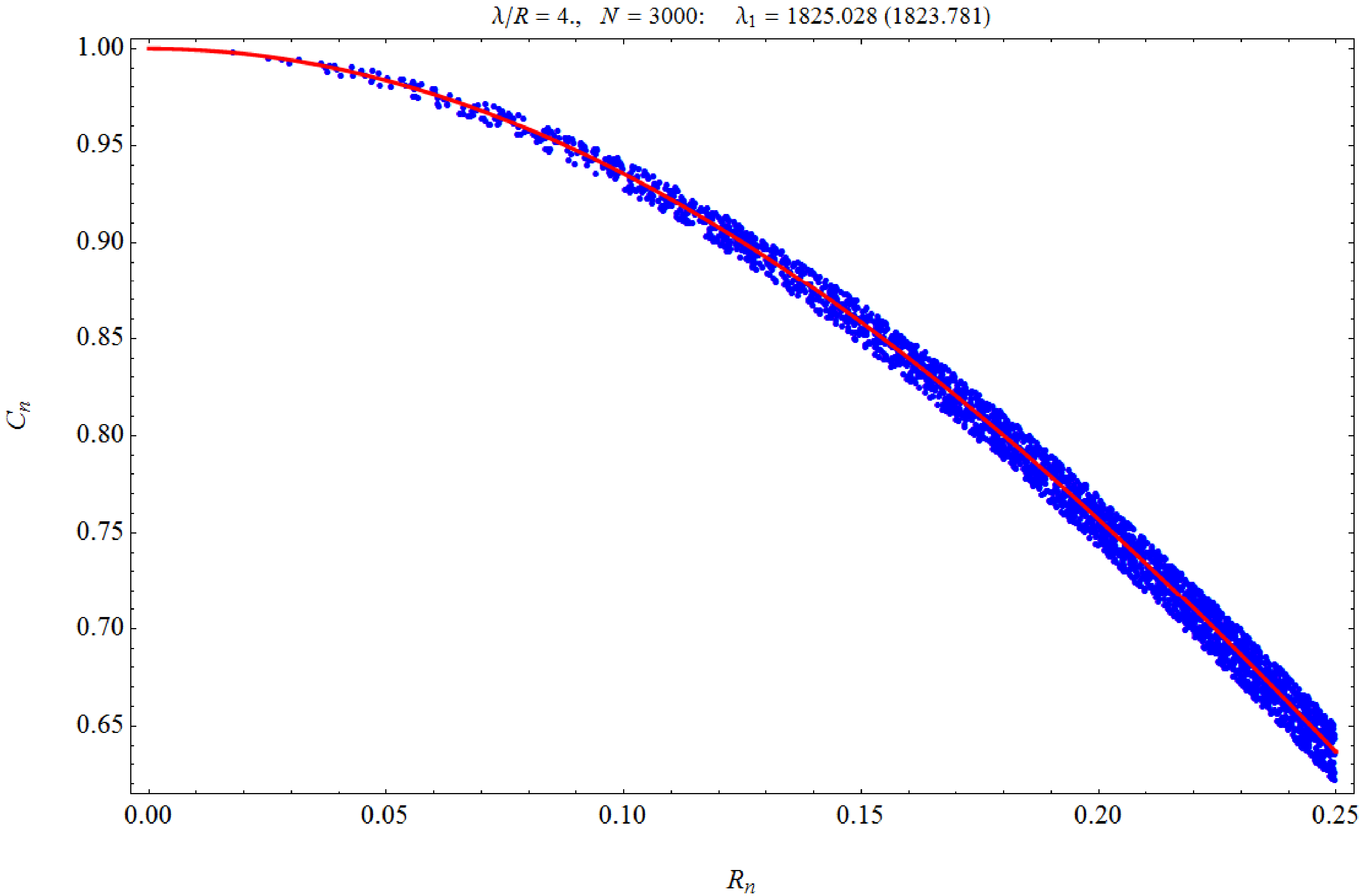
Dependence on shape, short wavelength



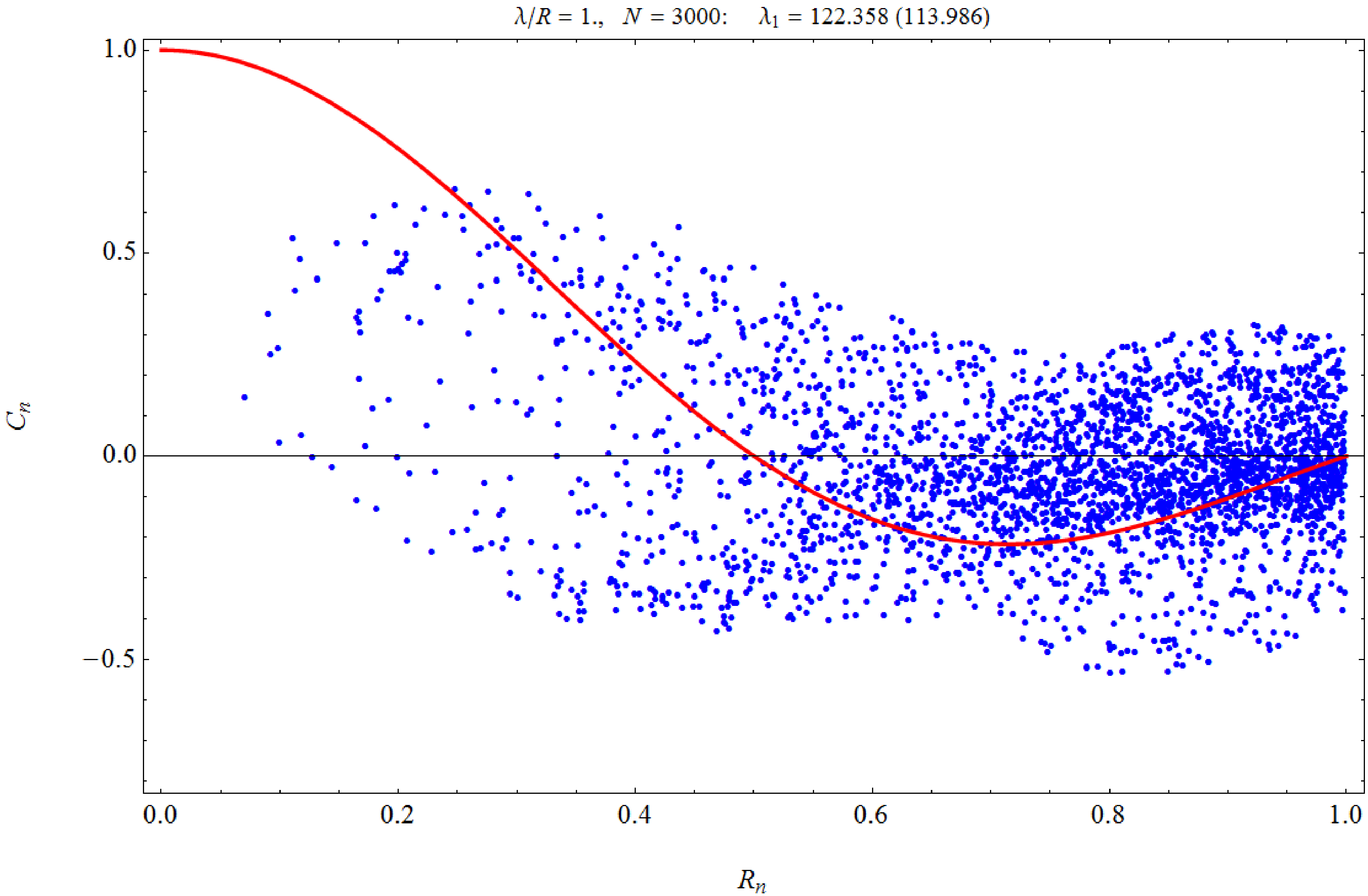
Superradiant state vector, large wavelength



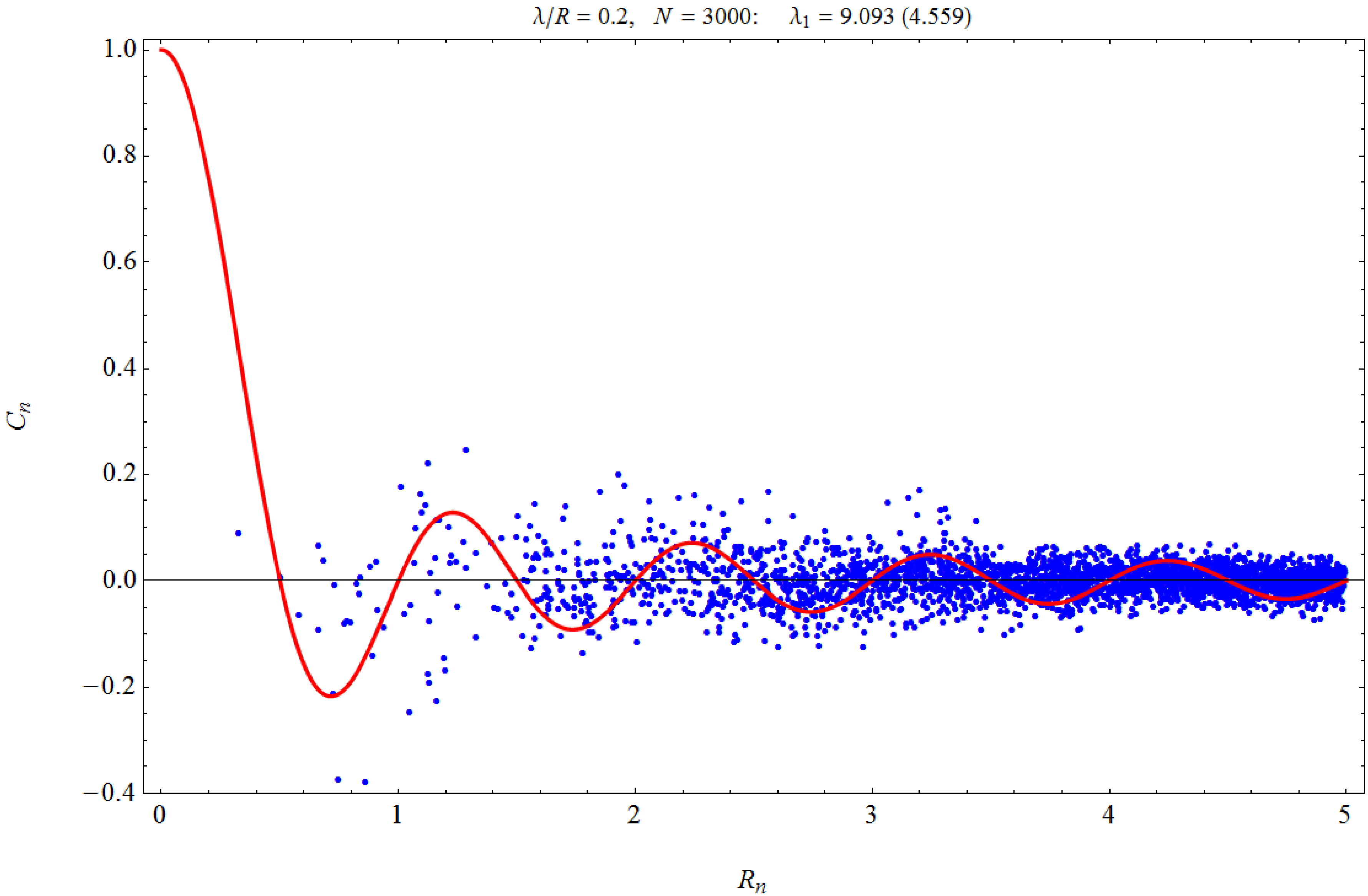
Superradiant state vector,
wavelength 4 times larger than size of sphere



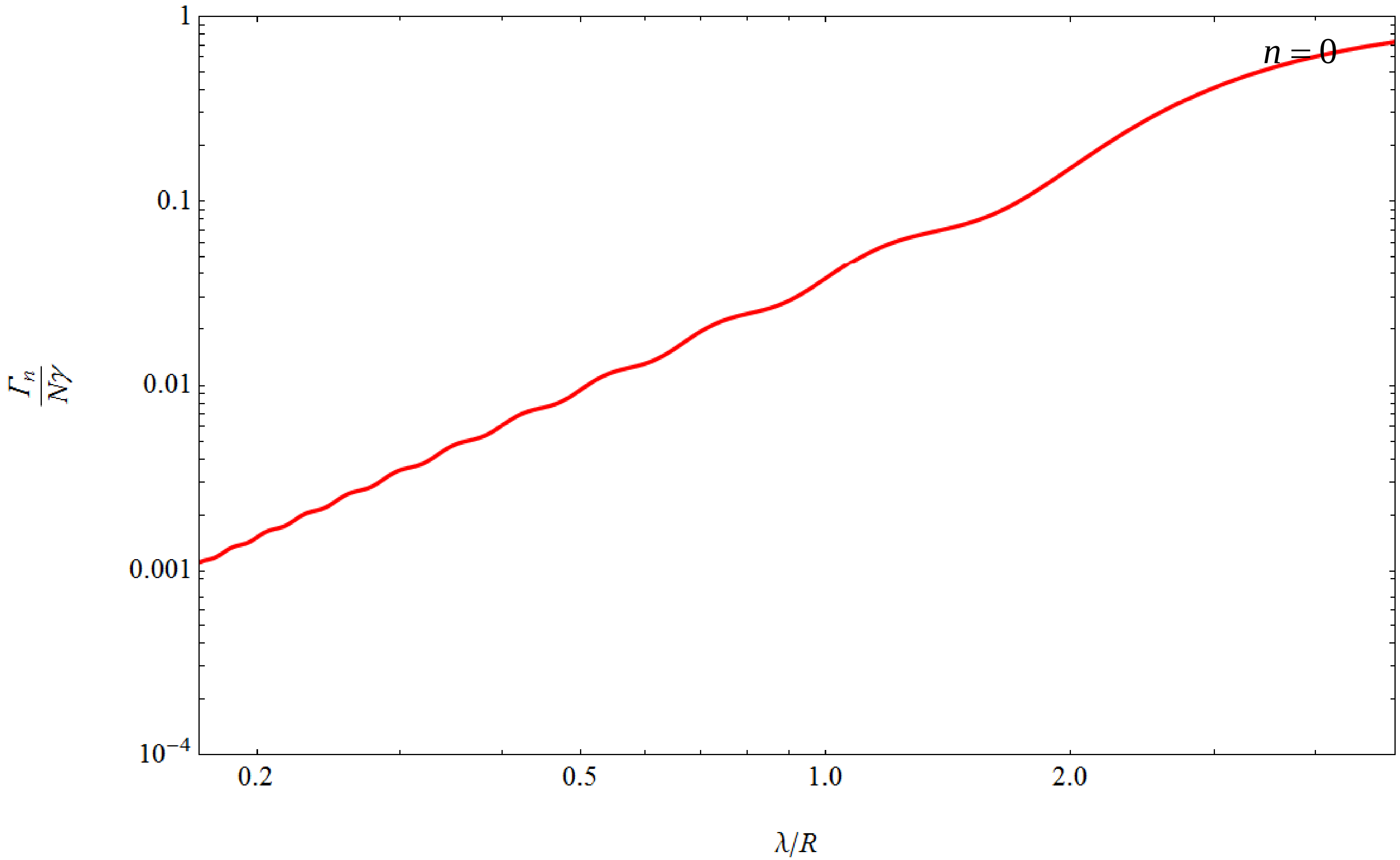
Superradiant state vector, wavelength equal to radius of sphere



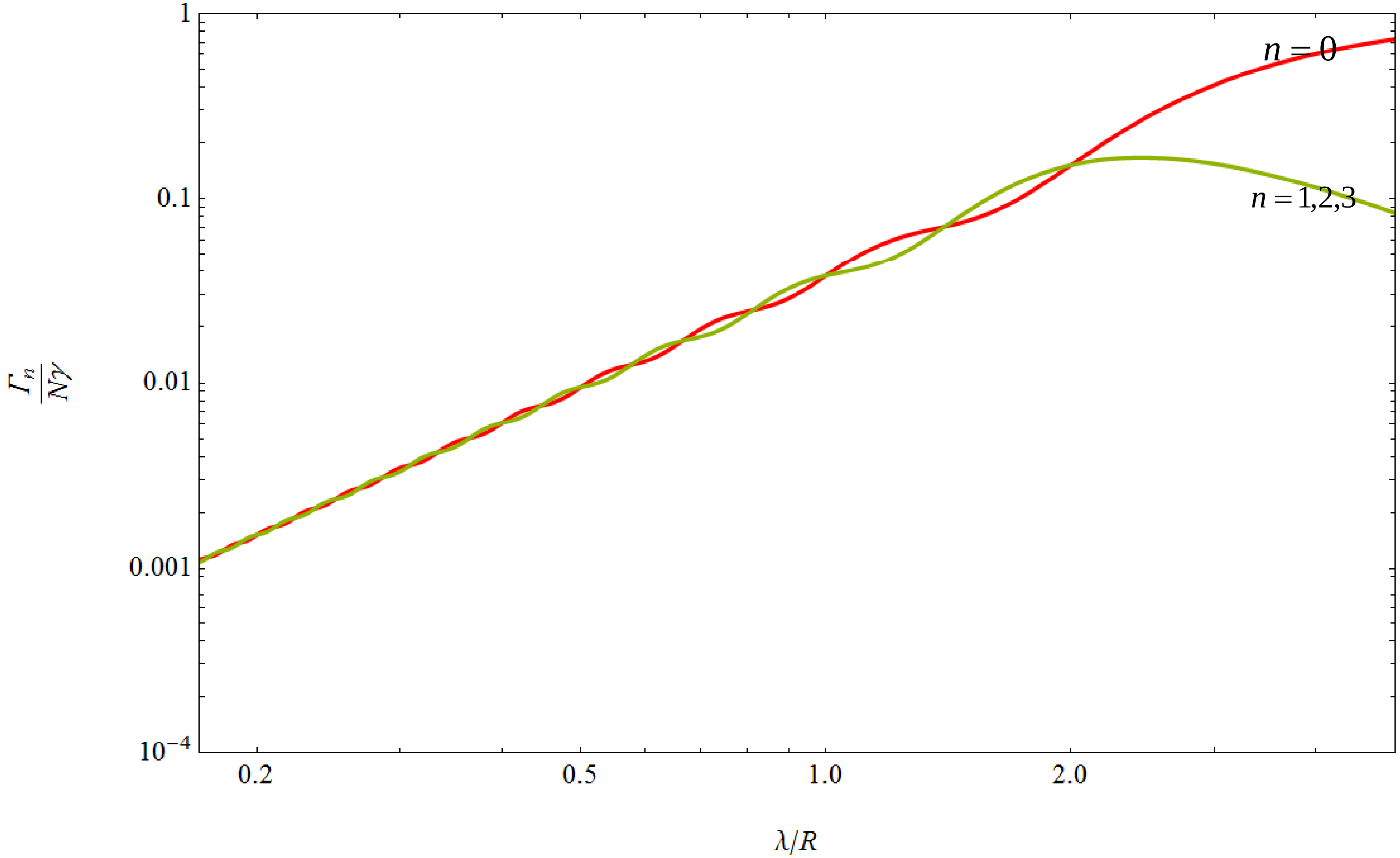
Superradiant state vector, small wavelength



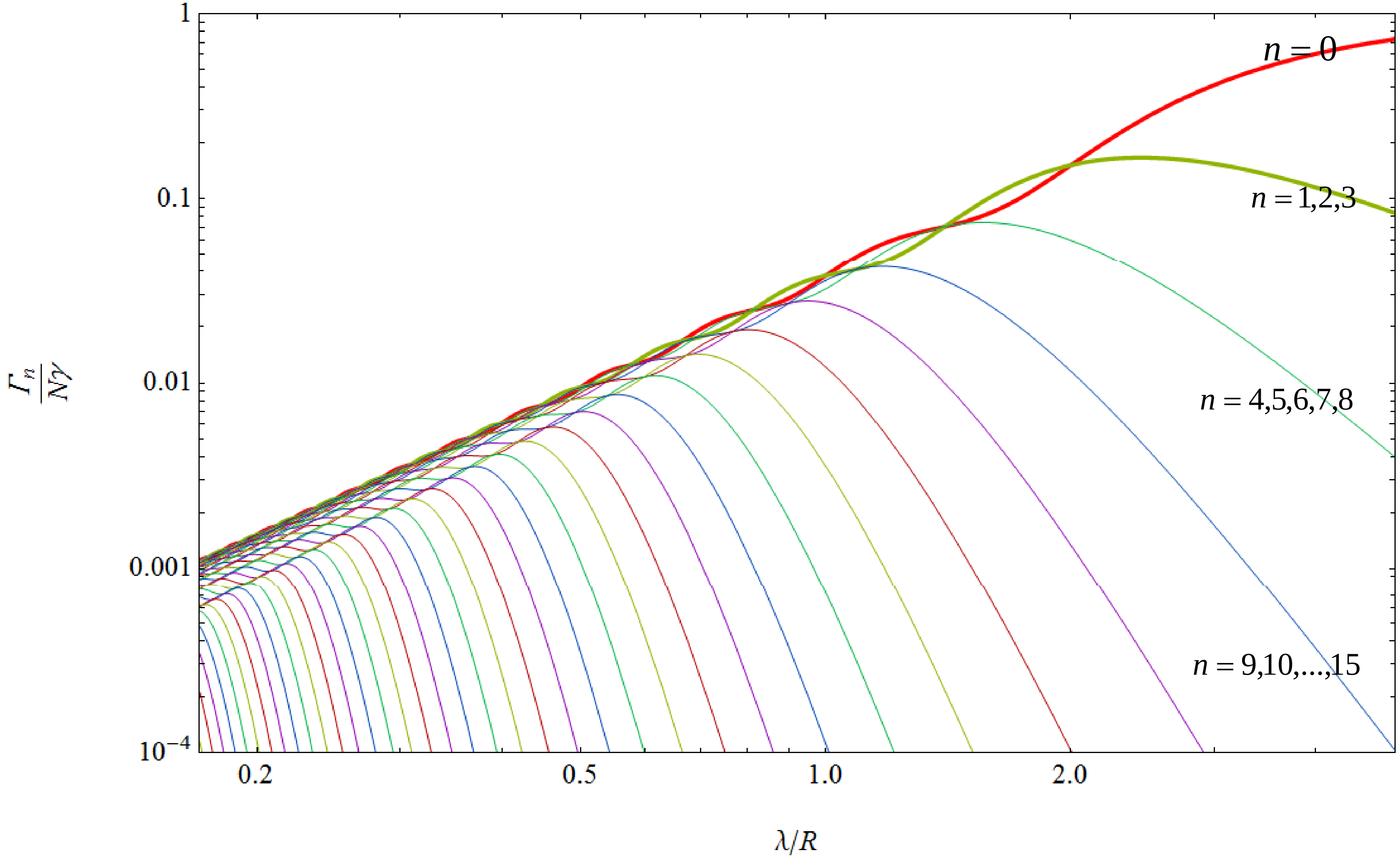
Decay rates for different wavelength



Decay rates for different wavelength



Decay rates for different wavelength



Calculating the decay rate of “superradiance” Γ_0

$$\Gamma_0^{(N\to\infty)} = \frac{3N}{2(k_0R)^2}\left[1-\frac{\sin(2k_0R)}{2k_0R}\right]$$

$$\boldsymbol{\beta}_{n,m} = \left\{j_n(k_0r_i)\,Y_{n,m}(\theta_i,\varphi_i)\right\}_{/\,i=1,2,3,\ldots,N}, \quad \mathbf{B} = \left\{\boldsymbol{\beta}_{n,m}\right\}_{/\,n\le n_{\max}}$$

$$\mathbf{F}(n_{\max}) = \mathbf{B}\mathbf{F}\mathbf{B}^{\mathrm{T}}, \quad \mathbf{S}(n_{\max}) = \mathbf{B}\mathbf{B}^{\mathrm{T}}$$

$$\Gamma_0(n_{\max}): \quad \text{largest zero of polynomial}\left|\mathbf{F}(n_{\max}) - \Gamma\mathbf{S}(n_{\max})\right|.$$

$$\text{Size of the matrix}: (n_{\max}-1)^2 \text{ instead of } N.$$

Numerical results for Γ_0

$N=1000,\ \lambda/R=1$	
$\Gamma_0^{(N\to\infty)}$	38.00
$\Gamma_0(0)$	39.90
$\Gamma_0(1)$	44.05
$\Gamma_0(2)$	44.19
$\Gamma_0(3)$	45.96
$\Gamma_0(4)$	46.39
$\Gamma_0^{(N)}$	46.55

$N=1000,\ \lambda/R=1/4$	
$\Gamma_0^{(N\to\infty)}$	1.52
$\Gamma_0(0)$	2.48
$\Gamma_0(1)$	3.21
$\Gamma_0(2)$	3.28
...	...
$\Gamma_0(10)$	4.30
$\Gamma_0^{(N)}$	4.63